(D) $2x - 4$

(E) $20x$

MATRICES

Matrices are a way of presenting information. Matrix questions are rare on the SAT Subject Test in Math 2, but there are a few different things which ETS may ask you to do.

Determinants of Matrices

The determinant of the 2 × 2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $\boldsymbol{ad - bc}$.

A Clue
A good way to remember the determinant of a 2 × 2 matrix is that you multiply the diagonals and subtract.

The determinant of a matrix is sometimes indicated by plain vertical bars around the elements, like a big absolute value symbol. The folks at ETS may simply write $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ if they want you to find the determinant of the matrix above.

The determinant of the 3×3 matrix

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \text{ is } aei + bfg + cdh - bdi - afh - ceg.$$

If you take the first two columns of the matrix and recopy them to the right of the original matrix, the parts of the formula form diagonal lines of three elements, with the positive parts going from the upper left to the bottom right, and the negative parts going from the upper right down to the bottom left, like this:
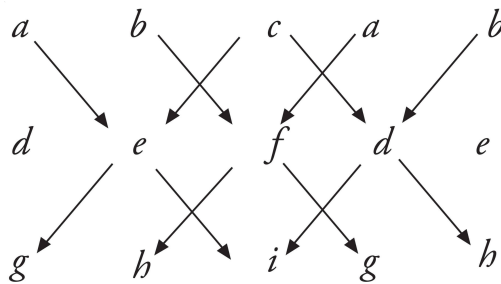
A What by What?

When you're looking at a particular matrix, for example, a 3 × 2 matrix, remember that this describes the matrix as row by column. So there are 3 rows and 2 columns.

Matrix Multiplication

The simplest sort of multiplication you may be asked to do with a matrix is to multiply a matrix by a single term. In these cases, the product is a matrix the same size as the original matrix, and each position is multiplied by the single term.

For example:

$$3\begin{bmatrix} 2 & 1 \\ 4 & 2 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 3\times 2 & 3\times 1 \\ 3\times 4 & 3\times 2 \\ 3\times 0 & 3\times 5 \end{bmatrix} = \begin{bmatrix} 6 & 3 \\ 12 & 6 \\ 0 & 15 \end{bmatrix}$$

The more complicated type of matrix multiplication involves multiplying two matrices together. Unlike other types of multiplication in math, order matters when you multiply matrices; i.e. matrix A × matrix B won't typically equal matrix B × matrix A. You can only multiply matrices when the first matrix has the same number of columns as the second matrix has rows. The product matrix will have the same number of rows as the first matrix and the same number of columns as the second matrix.

To illustrate:

$$\begin{bmatrix} - & - \\ - & - \\ - & - \end{bmatrix} \times \begin{bmatrix} - & - & - \\ - & - & - \end{bmatrix} = \begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \end{bmatrix}, \text{ but } \begin{bmatrix} - & - & - \\ - & - & - \end{bmatrix} \times \begin{bmatrix} - & - \\ - & - \end{bmatrix} = \begin{bmatrix} - & - \\ - & - \end{bmatrix}$$

Now, to determine what goes into the product matrix, you will need to follow some rules. The first row, first column position of the product matrix is the *dot product* of the first row of the first matrix and the first column of the second matrix. The dot product is the sum of the products of the row and column matched by position:

$$\begin{bmatrix} a & b & c \\ - & - & - \end{bmatrix} \times \begin{bmatrix} u & - \\ w & - \\ y & - \end{bmatrix} = \begin{bmatrix} (au + bw + cy) & - \\ - & - \end{bmatrix}$$

In other words, the first element in the row of the first matrix is multiplied by the first element in the column of the second matrix, the second element by the second, and the third by the third. Then you add them up, and that goes into the first row, first column of the product matrix.

The easy way to remember where each dot product ends up in your product matrix is that the **first row, first column** of your product is the **first row** of the first matrix multiplied by the **first column** of your second matrix. The **first row, second column** of the product will be the **first row** of the first times the **second column** of the second, and so on:

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \times \begin{bmatrix} u & v \\ w & x \\ y & z \end{bmatrix} = \begin{bmatrix} (au + bw + cy) & (av + bx + cz) \\ (du + ew + fy) & (dv + ex + fz) \end{bmatrix}$$

Coefficient Matrices

Coefficient matrices are a way of expressing the information given in a system of linear equations. A system of linear equations is a group of equations where none of the variables have exponents (thus, all linear), and you have as many equations as variables.

Let's take an example:

$$3x - y + 2z = 21$$

$$y + 3 = z$$

$$2z + y = 12$$

First, rearrange the equations so each has all the terms with variables on the left side of the equals sign in alphabetical order, and the constants on the right side of the equation:

$$3x - y + 2z = 21$$

$$y - z = -3$$

$$y + 2z = 12$$

Now, you have 3 equations and 3 variables, so your coefficient matrix will be 3 × 3. Each row will represent one equation. The first column will be the x-value of each equation, the second the y-value, and the third the z-value:

$$\begin{bmatrix} 3 & -1 & 2 \\ 0 & 1 & -1 \\ 0 & 1 & 2 \end{bmatrix}$$

Note that if an equation does not have a particular variable, that value is a 0 in the matrix. Terms without coefficients are understood to have a coefficient of 1.

You may in rare occasions be asked to express the system of equations using matrices. To do so, you only need add two simple things to your coefficient matrix: the variable matrix and the constant matrix.

The variable matrix is simply your variables listed in order in a column. You multiply the variable matrix by the coefficient matrix:

$$\begin{bmatrix} 3 & -1 & 2 \\ 0 & 1 & -1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

The result of this multiplication would return you back to the left side of

the equations. The product of these two matrices will be equal to your constants from the original system of equations:

$$\begin{bmatrix} 3 & -1 & 2 \\ 0 & 1 & -1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 21 \\ -3 \\ 12 \end{bmatrix}$$

For the purposes of the SAT Subject Test in Math 2, this is all you need to know about matrices!

Matrices and Your Calculator

Many calculators can do operations on matrices. The first step for any matrix question will be to input the matrices in your calculator. On the TI-84, press 2ND->x^{-1} to access the MATRIX menu. The first submenu is titled NAMES; however, you need to start under the EDIT submenu.

The EDIT submenu looks identical to the NAMES submenu, but here you can, well, edit the matrices. To edit a matrix, first input the size of the matrix (rows by columns). Press ENTER after you finish inputting the size of the matrix. Use ENTER to move on to the next position after inputting the value. Use the QUIT function to leave this menu after you are finished.

To find the determinant of a matrix, go back to the MATRIX menu and go to the MATH submenu. The first option, det, is to find the determinant. Select that option. Next, go back to the MATRIX menu and under NAMES select the matrix you wish to find the determinant of. Close the parentheses, press ENTER, and voila!

To multiply matrices, simply enter both matrices in the EDIT submenu, select the first matrix under NAMES, use the normal multiplication key, select the second matrix, and press ENTER. Your calculator will do all the hard work of determining dot products for you.

DRILL 12: MATRICES

If you feel ready for the matrix, try these examples. The answers can be found in Part IV.

30. If matrix X has dimension 3×2, matrix Y has dimension 2×5, and $XY = Z$, then matrix Z must have dimension

    (A) 2×2
    (B) 2×5
    (C) 3×2
    (D) 3×5
    (E) 6×10

40. If $A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$, then what is the determinant of A ?

    (A) -2
    (B) -1
    (C) 0
    (D) 1
    (E) 2

45. If $\begin{bmatrix} 0 & 1 & 3 \\ -1 & 1 & 2 \\ -2 & 1 & 2 \end{bmatrix} = X$, then $|X| =$

    (A) -2
    (B) 0
    (C) 1
    (D) 2
    (E) 3

$$2x + 3y - z = 12$$

$$x - 3y + 2z = -5$$

$$x + z = 3$$

46. What is the determinant of the coefficient matrix of the system of equations shown above?

(A) −6
(B) 0
(C) 2
(D) 3
(E) 10