Linear Equations (Algebraic) Answers

1. D
2. -2.5 or $-\frac{5}{2}$
3. B
4. D
5. A
6. C
7. 16
8. D
9. A
10. D
11. B
12. 1
13. A
14. $\frac{6}{7}$ or .857
15. $\frac{3}{2}$ or 1.5
16. C
17. $\frac{1}{2}$ or .5
18. B
19. 45
20. 4
21. 24
22. 15

Linear Equations (Algebraic) Explanations

1. **D.** This question is extremely basic. We know that in the Linear Equation $y = mx + b$, the value of m is the slope. To find a line with a slope of 3, we need to pick the only answer choice that includes the term "$3x$", which is Choice D.

2. **-2.5** or $-\frac{5}{2}$. We're just warming up, for now. To calculate slope between two points, we use the formula $\frac{y_2 - y_1}{x_2 - x_1}$.

Go ahead and plug in the coordinates from our two points. Remember, with this formula it doesn't matter which point you designate as "point 1" or "point 2," just remember to remain *consistent*. To be safe, it always helps to label or write down which coordinates you're calling "point 1" and "point 2" before you plug them into the equation.

Here's how I'll do it:

Point 1: $(-2,1)$ and Point 2: $(-1,-1.5)$

$$\begin{aligned}
&\frac{y_2 - y_1}{x_2 - x_1} \\
&= \frac{(-1.5)-(1)}{(-1)-(-2)} \\
&= \frac{-2.5}{-1+2} \\
&= \frac{-2.5}{1} \\
&= -2.5
\end{aligned}$$

So, the slope of this line is "-2.5" or, if you prefer, $-\frac{5}{2}$.

3. **B.** For this question, we need to provide a complete equation of the form $y = mx + b$. That means we'll need to know both the slope and the *y*-intercept.

Let's start with the slope, since we can easily calculate that from the two points that we've started with. Use the equation $\frac{y_2 - y_1}{x_2 - x_1}$. I'll use $(-1, .5)$ as Point 1 and $(1, 3.5)$ as point 2.

$$\begin{aligned}
&\frac{(3.5)-(.5)}{(1)-(-1)} \\
&= \frac{3}{1+1} \\
&= \frac{3}{2}
\end{aligned}$$

OK, good. Now we know that the slope of this line is 1.5 or $\frac{3}{2}$. But how to find the *y*-intercept?

The trick is to set up our $y = mx + b$ with the information we currently have, then plug in either one of the two (x, y) points given on the graph. I'll plug in the point $(1, 3.5)$:

$$\begin{aligned}
y &= mx + b \\
y &= \frac{3}{2}x + b \\
3.5 &= \frac{3}{2}(1) + b
\end{aligned}$$

Now solve for b, the *y*-intercept value:

$$\begin{aligned}
3.5 &= \frac{3}{2}(1) + b \\
3.5 &= 1.5 + b \\
-1.5 & \quad -1.5 \\
2 &= b
\end{aligned}$$

With our slope and *y*-intercept calculated, we can now finish our $y = mx + b$ Linear Equation:

$$\begin{aligned}
y &= mx + b \\
y &= \frac{3}{2}x + 2
\end{aligned}$$

4. **D.** For this question, we have two options. First, we could calculate the slope from the given x and y values, then use the $y = mx + b$ equation to solve for our *b*-value.

But to be honest, it will be faster to test the answer choices by plugging in values from the table into the provided answer choices, and eliminating an answer choice whenever it doesn't work.

Let's start by plugging in $x = 0$. The table tells us that any resulting *y*-value must equal 1.

If you plug in to Choice A, you'll get $y = 1$ - which is what we wanted - so we'll keep this option. Plug 0 into Choice B and you get -6, which isn't what we want. Plug into Choice C and you get -1, which isn't what we want either. However, when we plug into Choice D, we also get $y = 1$, which is the correct value.

Now we know that only Choices A and D are possibly correct. Let's pick another pair of values from the table and try again. I'll use $x = 1$, and the table says we're supposed to get $y = 4$ as a result.

In Choice A, when I plug in 1, I get -1, which isn't what we want. When I plug into Choice D, I get 4, which is what I'm looking for. By process of elimination, Choice D must be correct.

In this type of question, it's faster and easier just to test the answer choices. The reason I used $x = 0$ and $x = 1$ is that these are very quick and simple values to plug into the equations given in the answer choices.

5. **A.** To give the equation of a line, we need to create a $y = mx + b$ equation. To do that, we need the slope and the *y*-intercept. In this case, it's easiest to find the slope first. We can calculate slope using $\frac{y_2 - y_1}{x_2 - x_1}$. Let's set that up:

$$\frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{7-2}{0-(-5)}$$
$$= \frac{5}{5}$$
$$= 1$$

Now we know the slope of the line is 1. The next step is to set up everything we know about this line's equation, plug in one of the two points we're given, and calculate the *b*-value (the *y*-intercept) from that setup.

I'll use the point $(-5, 2)$ for x and y, but you can also use the point $(0, 7)$ if you want:

$$y = 1x + b$$
$$2 = 1(-5) + b$$
$$2 = -5 + b$$
$$+5 \quad +5$$
$$7 = b$$

Now we know our *b*-value, and can finish setting up our $y = mx + b$ equation:

$$y = mx + b$$
$$y = 1x + 7$$

Notice that the answer choices have been slightly rearranged, but it only takes a few steps of **Basic Algebra 1** to realize that Answer Choice A is the same equation as the one we're looking for.

6. **C.** This Linear Equation question focuses on a pair of "perpendicular" lines. Remember that the slope values will be *negative reciprocals* of each other. Therefore, if we know the slope of one line, we can easily calculate the slope of the other line.

The first thing to do is take the given equation and rearrange it into a more useful format. Let's do some **Basic Algebra 1** to convert the given $15x - 5y = -10$ equation into $y = mx + b$ format. This will tell us the slope of the first line:

$$15x - 5y = -10$$
$$-15x \qquad -15x$$
$$-5y = -15x - 10$$
$$\frac{-5y}{-5} = \frac{-15x - 10}{-5}$$
$$y = 3x + 2$$

OK, wonderful. We can see that the slope of the original line is 3. Now, we can find the slope of the perpendicular line with the negative reciprocal - the negative, upside-down version of 3 - which is $-\frac{1}{3}$.

Now, STOP for a second. Do you see how Choice B gives $-\frac{1}{3}x$ as part of the answer? It's a *trap*. None of these Answer Choices are currently in $y = mx + b$ form, which means they can't be trusted until we rearrange the equations into the proper format.

So, now's the time to go down the list of answer choices, rearranging them each into $y = mx + b$ form, and looking for the one line with a slope of $-\frac{1}{3}$.

Here are my conclusions. Choice A rearranges to $y = 3x + 4$, which does *not* have a slope of $-\frac{1}{3}$. Choice B rearranges to $y = \frac{1}{3}x + 4$, which does *not* have a slope of $-\frac{1}{3}$. Choice C rearranges to $y = -\frac{1}{3}x + \frac{4}{3}$, which *does* have a slope of $-\frac{1}{3}$. Choice D rearranges to $y = -3x + 4$, which does *not* have a slope of $-\frac{1}{3}$.

Note that, throughout our reasoning, we did *not* put any focus on the *y*-intercept value. That's because *y*-intercepts are not related to perpendicular lines; perpendicular lines are only about slope.

7. **16.** OK, this question has a lot in common with Question 4. However, this time we aren't given any multiple choice answers, so we'll have to actually figure out the $y = mx + b$ Linear Equation for this table.

We can treat $f(x)$ values exactly like *y*-values (study the upcoming lesson on **Functions** for more on this topic). So, where the table says $f(x)$, just imagine it says "y" instead.

Let's start by calculating the slope, since we have several coordinates to draw from. Use the $\frac{y_2 - y_1}{x_2 - x_1}$ equation. I'll select $(3,9)$ as my Point 1, and $(-5,-7)$ as my Point 2.

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{(-7)-(9)}{(-5)-(3)} = \frac{-16}{-8} = 2$$

Alright, now we've found that this linear function has a slope of 2.

Now use the $y = mx + b$, plug in the slope of 2, and then plug in any of the three (x, y) coordinates given in the table. I'll use $(3,9)$.

$$\begin{aligned} y &= mx + b \\ y &= 2x + b \\ 9 &= 2(3) + b \\ 9 &= 6 + b \\ -6 & \;\; -6 \\ 3 &= b \end{aligned}$$

Great - now we know the *y*-intercept and can complete our $y = mx + b$ equation:

$$y = 2x + 3$$

The final step is to evaluate $f(6.5)$. This means to plug $x = 6.5$ into our Linear Equation. Again, be sure to study the lesson on **Functions** if this notation form is confusing or unclear to you.

$$\begin{aligned} y &= 2x + 3 \\ y &= 2(6.5) + 3 \\ y &= 13 + 3 \\ y &= 16 \end{aligned}$$

8. **D.** This question provides us with a Linear Equation in the form $y = nx + 2$, and asks us to give the slope. Instead of using numbers, the Answer Choices are mainly based on variables. How to make sense of it?

The first step may seem strange to you: we're going to plug in the point (a,b) for our *x*- and *y*-values:

$$\begin{aligned} y &= nx + 2 \\ b &= na + 2 \end{aligned}$$

I know it may seem strange to replace two letters with two more letters, but the point (a,b) is on our line, which makes it valid to plug into our Linear Equation for x and y.

We've been asked to solve for the slope of the line. Now, in the original equation $y = nx + 2$, where do we find the slope of the line? It's the coefficient of x, of course - in this case, that would be the letter n.

So, we'll take our equation and solve for n:

$$\begin{aligned} b &= na + 2 \\ -2 &\quad\; -2 \\ b - 2 &= na \\ \frac{b-2}{a} &= \frac{na}{a} \\ \frac{b-2}{a} &= n \end{aligned}$$

And we're done. The slope of this line can be expressed as $\frac{b-2}{a} = n$, or Choice D.

By the way, the reason the question includes the warnings that $a \neq 0$ and $b \neq 0$ is simply to prevent any of our answer choices from dividing by zero, which is never allowed in math. It's basically info that we can ignore in this question.

9. **A.** This question tasks us with finding a "parallel" line to an existing line. Remember that *parallel* lines always have *equal* slopes. First, let's find the slope of the original line by putting it into $y = mx + b$ form:

$$\begin{aligned} -4x - 3y &= 12 \\ +4x \qquad &\quad +4x \\ -3y &= 4x + 12 \\ \frac{-3y}{-3} &= \frac{4x+12}{-3} \\ y &= -\frac{4}{3}x - 4 \end{aligned}$$

Now we can read the original line as having a slope of $-\frac{4}{3}$. So, our new line must also have the same slope. Luckily, it's easy to find: only Choice A has a slope of $-\frac{4}{3}$.

10. **D.** There are two options for solving this problem. The "low-tech" way is to basically count on your fingers and eliminate wrong answers. We start at the origin, point $(0,0)$, as given in the question. This eliminates Choice A, because if our line is already at $(0,0)$, it can't also be at $(0,9)$ at the same time. Then imagine tracking your line from left to right. Our line has a slope of $-\frac{9}{2}$, so it will go *down* 9 units for every 2 units we move to the right. That eliminates Choice B, which went *up* 9 units when we moved to the right by 2 units.

Now return to $(0,0)$ and start moving right to left. When we move left by 2 units, the line will go *up* by 9 units. That eliminates Choice C. But, if we move to the left by 4 units, the line will go up 18 units, which puts us right at point $(-4,18)$.

The other way to solve this question is using the $y = mx + b$ form. Plug in the values we know: slope is $-\frac{9}{2}$ and the line contains the point $(0,0)$ - which means the *y*-intercept is 0:

$$\begin{aligned} y &= mx + b \\ y &= -\frac{9}{2}x + 0 \\ y &= -\frac{9}{2}x \end{aligned}$$

Now you could test each of the answer choices by plugging them into this formula. If the point does not return a true equality when plugged in, then eliminate it. You'll again find that only Choice D returns a true equality.

11. **B.** Probably the easiest way to deal with this question is to play around with sketching a few possible graphs. First draw a simple *xy*-axis. Then place a point in each of Quadrants I, II, and III - but not in Quadrant IV (review the section of the lesson on Quadrants if you need a reminder of which is which).

Now try to sketch a single straight line that passes relatively close to each of your three points, without dipping into Quadrant IV at all. It may be difficult or impossible to make your straight line pass exactly through all three points, but that's OK. You're just using them as a guide for your sketch.

Within a couple of tries you'll realize that only lines with *positive slopes* can pass through these three quadrants.

12. **1.** Two parallel lines will always have equal slopes. Therefore, let's calculate the slope of the upper line, which has provided us with exact coordinates for two points. I'll use $(-3,-2)$ as Point 1 and $(-1,2)$ as Point 2:

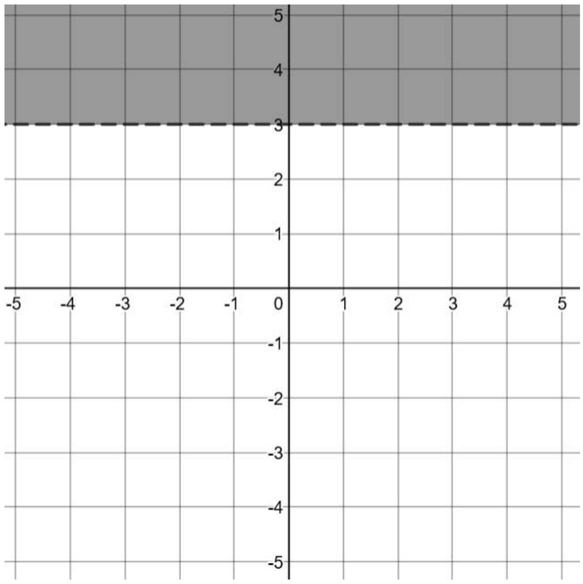
$$\begin{aligned}&\frac{y_2-y_1}{x_2-x_1}\\&=\frac{(2)-(-2)}{(-1)-(-3)}\\&=\frac{2+2}{-1+3}\\&=\frac{4}{2}\\&=2\end{aligned}$$

The slope of the upper line must be 2. OK, now we can set up another slope equation for the lower line. We know the value of this slope must also equal 2. We will also use the unknown *x*-coordinate r in our slope equation, since we haven't figured out its value yet. Then solve for r using **Basic Algebra 1**:

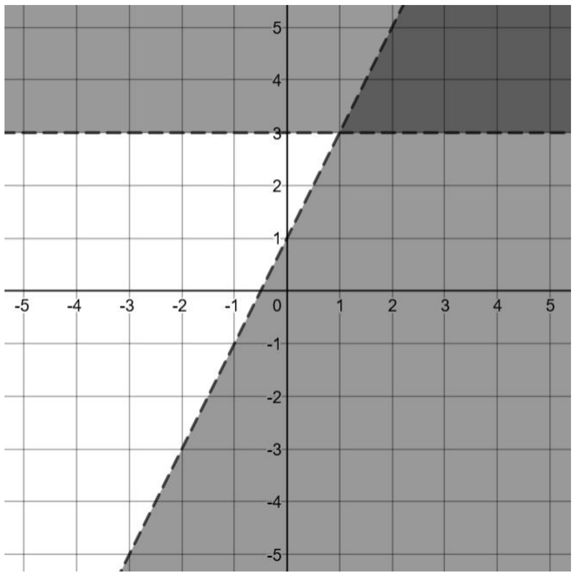
$$\begin{gathered}\frac{y_2-y_1}{x_2-x_1}=2\\\frac{(-2)-(-4)}{(r)-(0)}=2\\\frac{-2+4}{r-0}=2\\\frac{2}{r}=2\\(r)\frac{2}{r}=2(r)\\2=2r\\\frac{2}{2}=\frac{2r}{2}\\1=r\end{gathered}$$

There we go - the value of r must be 1.

13. **A.** The safest way to solve this problem is probably to actually draw it out. Sketch an *xy*-coordinate plane with around 5 tick marks per "arm" of the axes. It's easy to graph $y>3$. Simply draw a horizontal dotted line at $y=3$, then lightly shade everything above it, like this:



Next, we'll add the line for $y<2x+1$. This will have a *y*-intercept at 1, and a slope of 2. Sketch this line (it should be a dotted line), then shade *under* it. It looks something like this:



Almost done. Now that we've sketched the situation, we can find the region where *both* inequalities have shading. That's in the upper-right corner of our graph, in Quadrant I.

14. **.857** or $\frac{6}{7}$ **.** The first step of this question - like most others in this lesson - is to set up the $y = mx + b$ equation. But before we can do that, we have to determine the slope of the line. It may seem impossible at first, because there only seems to be the point $(5,3)$ that gives us complete values to work with.

Luckily, if you think carefully, we also have another point at $(-2,0)$. This is where the line "crosses the *x*-axis," which means it must have a *y*-coordinate of 0.

Let's calculate the slope using the points $(5,3)$ as Point 1 and $(-2,0)$ as Point 2.

$$\frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{(0)-(3)}{(-2)-(5)}$$
$$= \frac{-3}{-7}$$
$$= \frac{3}{7}$$

OK, now we know the slope is $\frac{3}{7}$:

$$y = \frac{3}{7}x + b$$

Now, remember that we're solving for g, which comes from the coordinates $(0,g)$. Notice that this point happens to be the *y*-intercept since the x-coordinate is 0, so we can plug g directly in for b:

$$y = \frac{3}{7}x + b$$
$$y = \frac{3}{7}x + g$$

Now, let's use our point $(5,3)$ to plug in for x and y, then solve for g. We'll also need our knowledge of **Fractions** to finish the subtraction in the last few steps.

$$y = \frac{3}{7}x + g$$
$$3 = \frac{3}{7}(5) + g$$
$$3 = \frac{15}{7} + g$$
$$-\frac{15}{7} - \frac{15}{7}$$
$$3 - \frac{15}{7} = g$$
$$\frac{21}{7} - \frac{15}{7} = g$$
$$\frac{6}{7} = g$$

15. **1.5** or $\frac{3}{2}$. Solving this question won't be as hard as it might look. We know that an *x*-intercept is where the line crosses the *x*-axis. Therefore, any *x*-intercept will always have a *y*-coordinate of 0. Let's represent this desired coordinate as $(X,0)$ where X represents the *x*-coordinate we're looking for. Now, let's try plugging this coordinate into the given equation for the line:

$$\frac{2}{3}x - \frac{5}{4}y = 1$$
$$\frac{2}{3}X - \frac{5}{4}(0) = 1$$
$$\frac{2}{3}X - 0 = 1$$
$$\frac{2}{3}X = 1$$
$$(\frac{3}{2})\frac{2}{3}X = 1(\frac{3}{2})$$
$$X = \frac{3}{2}$$

And we're done! This line crosses the *x*-axis at $(\frac{3}{2}, 0)$ and our final answer is $\frac{3}{2}$ or 1.5.

16. **C.** To solve the question, we have to have a crucial realization: although it seems like there are only two given points, there's actually a *third* point hiding within the words: this line passes through the origin, which means it also has the point $(0,0)$.

So, how can we use this? Instead of creating a $y = mx + b$, this time we'll focus only on the Slope equation $\frac{y_2 - y_1}{x_2 - x_1}$.

Since all three points are on the same line, the slope between all three points should be exactly the same. So, we can set up *two* Slope equations and set them equal to each other. Something like this:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_3 - y_1}{x_3 - x_1}$$

Notice that I've adjusted the right side of the equation to use a *third* point. Let's call $(-4, k)$ Point 1, we'll make $(k, -36)$ Point 2, and $(0,0)$ will be Point 3. Plus them all into the equation above and solve for k, using techniques we learned in the first two lessons on **Algebra 1**.

$$\frac{(-36)-(k)}{(k)-(-4)} = \frac{(0)-(k)}{(0)-(-4)}$$
$$\frac{-36-k}{k+4} = \frac{0-k}{0+4}$$
$$\frac{-36-k}{k+4} = \frac{-k}{4}$$
$$4(-36-k) = -k(k+4)$$
$$-144-4k = -k^2-4k$$
$$+k^2+4k \quad +k^2+4k$$
$$k^2-144=0$$
$$+144 \quad +144$$
$$k^2 = 144$$
$$\sqrt{k^2} = \sqrt{144}$$
$$k = 12$$

17. $\frac{1}{2}$ or **.5.** As always, the word "perpendicular" in the context of a Linear Equation question means that we'll use one slope to calculate the negative reciprocal slope of the perpendicular line.

So, first we'll need the original slope of function f, shown in the graph. It may seem low-tech, but we're just going to measure rise over run on the gridlines.

Notice the line for f passes directly through coordinates $(-1,1)$ and $(0,-3)$. We can calculate the slope by using the $\frac{y_2 - y_1}{x_2 - x_1}$ equation. Let's do so now:

$$\frac{(-3)-(1)}{(0)-(-1)}$$
$$= \frac{-3-1}{0+1}$$
$$= \frac{-4}{1}$$
$$= -4$$

Now we know that the slope of our original line for function f is -4. To calculate the slope of function g, we'll take the negative reciprocal of -4, which is $\frac{1}{4}$.

We're making progress. Now we'll set up a $y = mx + b$ Linear Equation for function g and plug in the slope, along with the point $(-2,1)$, which the question tells us is a point on the line for g. This will allow us to solve for the *y*-intercept or *b*-value of function g:

$$y = mx + b$$
$$y = \frac{1}{4}x + b$$
$$1 = \frac{1}{4}(-2) + b$$
$$1 = -\frac{1}{2} + b$$
$$+\frac{1}{2} \quad +\frac{1}{2}$$
$$\frac{3}{2} = b$$

Now we know both the slope and the *y*-intercept of the Linear Equation for function g, so let's put that all together:

$$y = mx + b$$
$$y = \frac{1}{4}x + \frac{3}{2}$$

And now, we can finish the question, which asks for the value of $g(-4)$. That means to plug in -4 for x into the equation for function g that we've just created (the lesson on **Functions** contains additional practice on this topic). Then, finish evaluating to get the final value of $g(-4)$:

$$y = \frac{1}{4}x + \frac{3}{2}$$
$$y = \frac{1}{4}(-4) + \frac{3}{2}$$
$$y = -1 + \frac{3}{2}$$
$$y = \frac{1}{2}$$

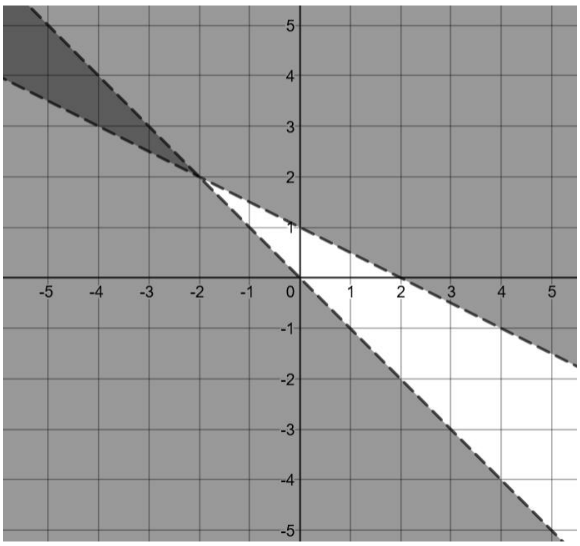
It may have taken a lot of work, but now we know the value of $g(-4)$ is $\frac{1}{2}$ - or in other words, when you plug $x = -4$ into the equation, you get $y = \frac{1}{2}$.

18. **B.** Like Question 13, the easiest thing to do here is actually graph the two inequalities. By this point you should be comfortable graphing equations based on $y = mx + b$.

The first line has a *y*-intercept of 0 and a slope of -1. It will be a dotted line, with shading *underneath* it.

The second line has a *y*-intercept of 1, and a slope of $-\frac{1}{2}$. It will also be a dotted line, with shading *above* it.

The result looks like this:



As you can see, the only solutions (where both shaded regions overlap) are in Quadrant II.

19. **45.** As with all Linear Equations questions involving a "perpendicular" line, we'll need to calculate the slope of the first line and then take the negative reciprocal to get the slope of the perpendicular line. Let's use the equation $y + 2 = \frac{3}{2}x$ and put it into $y = mx + b$ form:

$$y + 2 = \frac{3}{2}x$$
$$-2 \qquad -2$$
$$y = \frac{3}{2}x - 2$$

Now we can see that the slope of the first line is $\frac{3}{2}$. That means the perpendicular line t will have a slope of $-\frac{2}{3}$.

Next we'll set up a $y = mx + b$ equation for line t.

$$y = mx + b$$
$$y = -\frac{2}{3}x + b$$

We also know that line t passes through the x-intercept of -40, which has the coordinates $(-40, 0)$. We can plug these coordinates into the equation for line t, then solve for the *y*-intercept or *b*-value:

$$y = -\frac{2}{3}x + b$$
$$0 = -\frac{2}{3}(-40) + b$$
$$0 = \frac{80}{3} + b$$
$$-\frac{80}{3} \quad -\frac{80}{3}$$
$$-\frac{80}{3} = b$$

It's an ugly fraction, but now we know the *y*-intercept of line t is $-\frac{80}{3}$. Put this together with the slope to complete our Linear Equation for line t.

$$y = -\frac{2}{3}x - \frac{80}{3}$$

Last but not least, finish the question by plugging in the point $(27.5,-n)$:

$$-n=-\frac{2}{3}(27.5)-\frac{80}{3}$$
$$-n=-\frac{55}{3}-\frac{80}{3}$$
$$-n=-\frac{135}{3}$$
$$-n=-45$$
$$n=45$$

20. **4.** This question is an interesting sort of challenge that you only really see on the SAT Math test. I had never seen a question like this in my high school classes, for example - and most of my students feel the same way. Now, it does also include a System of Equations, which means there's more than one way to solve it (you can study the upcoming lesson on **Systems of Equations** for more information).

But here's why this question relates to Linear Equations, and why I think it belongs in this chapter.

Now, follow me through on some logic. When you graph two Linear Equations, what does their solution look like on the graph?

It's the point where they intersect, right?

So, if two Linear Equations have *no* solution, what does that mean? It means that they *never* intersect.

And how do you get two lines that never intersect?

That's right, they must be *parallel* to each other.

And if two lines are parallel, what else do you know about them?

Yup - they have the exact same slopes.

So, what we'll do is put both of the given equations into $y=mx+b$. Then we'll take their slopes and set them equal to each other, and see what happens.

First, let's get them both into $y=mx+b$ form.

$$\begin{aligned} 2x+y&=6\\ -2x\quad&\quad-2x\\ y&=-2x+6 \end{aligned}$$

$$\begin{aligned} 8-2y&=ax\\ -8\quad&\quad-8\\ -2y&=ax-8\\ \frac{-2y}{-2}&=\frac{ax-8}{-2}\\ y&=-\frac{a}{2}x+4 \end{aligned}$$

Now, compare the slopes of these two equations. The first one has a slope of -2. The second one has a slope of $-\frac{a}{2}$. Set those two slopes equal to each other, since the lines are parallel, then solve for a:

$$-2=-\frac{a}{2}$$
$$(-2)-2=-\frac{a}{2}(-2)$$
$$4=a$$

There we go: the value of a is 4.

Remember the mental process and tricks we used for this question with "no solutions"! This type of question *is* tested on the SAT - and like I said, most students have never seen it before. But, it's quite simple when you realize how "no solutions" relates to Linear Equations, parallel lines, and equal slopes.

21. **24.** This is a multistage Linear Equation problem, but it uses all the same concepts we've been practicing throughout this lesson.

We're being asked for a value of $g(15)$, which is a function - so, this overlaps a bit with the upcoming lesson on **Functions**. But, we can easily handle it with Linear Equations if you understand that $g(15)$ just means "plug $x=15$ into the equation for g."

Before we can do that, we'll need to actually *find* the equation for g. Since it's a linear equation, we'll have to find the slope and the *y*-intercept.

To find the slope for function g, we'll have to use "3 times the slope of the graph of f." But that's easy - we can just use the given graph to pick two points from the graph and calculate the slope of f.

Let's use the points $(0,-1)$ and $(3,0)$ since the line goes cleanly through those points:

$$\frac{(0)-(-1)}{(3)-(0)}$$
$$=\frac{1}{3}$$

The slope of line f is $\frac{1}{3}$. Since the slope of line g is "3 times that", we know that the slope of line g must be $3(\frac{1}{3})$, which equals a slope of 1.

Now, how to find the *y*-intercept for g ? Let's set up the $y = mx + b$ for function g with as much information as we have:

$$y = mx + b$$
$$y = 1x + b$$

The question also gives us an (x, y) coordinate $(-3, 6)$ that g passes through. Plug that in for x and y :

$$\begin{aligned} y &= 1x + b \\ 6 &= 1(-3) + b \\ 6 &= -3 + b \\ +3 &\quad +3 \\ 9 &= b \end{aligned}$$

OK, now we know the *y*-intercept for function g is 9, and the slope is 1, so we can finish our $y = mx + b$ setup for g :

$$y = 1x + 9$$

Now finish the question off by plugging in 15 for x , since we were asked for $g(15)$:

$$\begin{aligned} y &= 1x + 9 \\ y &= 1(15) + 9 \\ y &= 15 + 9 \\ y &= 24 \end{aligned}$$

Finished! The value of $g(15)$ is 24.

22. **15.** Compare this question to Question 20. It's almost exactly the same, with one difference: instead of having "*no* solutions," this system has "*infinitely many* solutions." Let's think this through one more time.

What does the solution to a pair of lines look like on a graph?

Right, it's the intersection point where both lines cross.

What does it mean if a pair of lines have *infinitely many* solutions?

It means they cross at infinite points!

How do two lines cross at infinite points? Well, only if they're the *exact same line*.

Therefore, these two equations must be the same line with the same $y = mx + b$ equations.

First, let's put them both into $y = mx + b$ form:

$$\begin{array}{ccc} tx + y = 5 & \qquad & -c = y - 5x \\ -tx \quad\quad -tx & & +5x \quad\quad +5x \\ y = -tx + 5 & & 5x - c = y \end{array}$$

So, these two equations are the same as each other. Set them equal:

$$-tx + 5 = 5x - c$$

Now compare them. They must have the same slopes. Therefore, $-t = 5$, so $t = -5$.

Since they're the same line, they must also have the same *y*-intercepts. Therefore, $5 = -c$, so $c = -5$.

The question asked for the value of $t - 4c$. If we plug in our values, we get:

$$\begin{aligned} & t - 4c \\ &= (-5) - 4(-5) \\ &= -5 + 20 \\ &= 15 \end{aligned}$$

Finished! $t - 4c$ must equal 15.

Remember, if two Linear Equations have "*no* solutions", they must be parallel lines and have the same slopes. If two Linear Equations have "*infinitely-many* solutions", they must be the *exact same lines*, with the same slopes *and* the same *y*-intercepts.

Systems of Equations Answers

1. C
2. B
3. D
4. A
5. D
6. D
7. 8
8. 9
9. B
10. B
11. D
12. 2750
13. A
14. A
15. 710
16. 28
17. 8
18. .5
19. D
20. C
21. C
22. 120
23. C
24. B
25. B
26. 7.5
27. A
28. A
29. A
30. D
31. C
32. A
33. D
34. .5 or $\frac{1}{2}$
35. 6
36. 120
37. A
38. 2 or 8
39. C
40. C
41. B
42. 5
43. B
44. A
45. D
46. C
47. A

Systems of Equations Explanations

1. **C.** This question is a basic System of Equations that already starts in Algebraic form. I'll use the ISS Method. You can also use the Elimination Method.

First, Isolate a variable. I'll Isolate y from the bottom equation:

$$\begin{aligned} 2y - 4x &= 0 \\ +4x \quad &+4x \\ 2y &= 4x \\ \frac{2y}{2} &= \frac{4x}{2} \\ y &= 2x \end{aligned}$$

Now that we've isolated y, we can plug it into the second equation.

$$\begin{aligned} 3x + 5y &= 26 \\ 3x + 5(2x) &= 26 \end{aligned}$$

Now distribute, combine like terms and Solve for x:

$$\begin{aligned} 3x + 5(2x) &= 26 \\ 3x + 10x &= 26 \\ 13x &= 26 \\ \frac{13x}{13} &= \frac{26}{13} \\ x &= 2 \end{aligned}$$

Now we know that $x = 2$. However, the question asked for the value of $x + y$, so we still need to find y by plugging $x = 2$ back into either of the original equations.

I'll just use the top equation for this:

$$\begin{aligned} 3x + 5y &= 26 \\ 3(2) + 5y &= 26 \\ 6 + 5y &= 26 \\ -6 \qquad &-6 \\ 5y &= 20 \\ \frac{5y}{5} &= \frac{20}{5} \\ y &= 4 \end{aligned}$$

Now we know that $y = 4$ and we can finish the final question:

$$\begin{aligned} &x + y \\ &= 2 + 4 \\ &= 6 \end{aligned}$$

And our final answer is 6, or Choice C.

2. **B.** There are a few ways to solve this System of Equations. Either the "ISS" Method or the Elimination Method will work equally well. However, regardless of the method I use, it seems smart to multiply the bottom equation by "2" before doing anything else - because it's nice to get rid of that fraction $\frac{y}{2}$:

$$\begin{aligned} (2)(\frac{y}{2} + 3x &= 23) \\ y + 6x &= 46 \end{aligned}$$

Now personally, I'm going to use the "ISS" Method to move on from here, because the top equation has already Isolated $y = x - 10$ for us. I can make a Substitution into the bottom equation that I just multiplied by 2:

$$\begin{aligned} y + 6x &= 46 \\ (x - 10) + 6x &= 46 \end{aligned}$$

Now I can finish the **Basic Algebra 1** for the value of x:

$$\begin{aligned} (x - 10) + 6x &= 46 \\ 7x - 10 &= 46 \\ +10 \quad &+10 \\ 7x &= 56 \\ \frac{7x}{7} &= \frac{56}{7} \\ x &= 8 \end{aligned}$$

OK, now we know that $x = 8$. And luckily, we can stop here - there's only one answer, Choice B, that gives an *x*-coordinate of 8.

If you wanted to be *really* sure, you could double-check by plugging $x = 8$ back into one of the original equations and confirm that $y = -2$ (it does, but check if you want).

3. **D.** This is another basic Systems of Equations question. My instinct is to solve it with the Elimination Method, rather than "ISS". I know in advance that if I start dividing by "3s" and "4s" to Isolate a variable, I'm going to get some ugly fractions, and I'd rather just not deal with them.

Instead, I'll multiply the top equation by 4 and the bottom equation by 3. In doing so I'll end up with the same number of n's on top and bottom:

$$(4)(3n+4t=29)$$
$$(3)(4n+3t=27)$$

$$12n+16t=116$$
$$12n+9t=81$$

On the bottom you can see my result after multiplying both equations and that the n terms are the same on top and bottom. Now I can subtract the bottom equation from the top equation to Eliminate all my n terms:

$$\begin{aligned} 12n+16t&=116 \\ -(12n+9t&=81) \\ 0n+7t&=35 \end{aligned}$$

And now I can solve for t quite easily:

$$\frac{7t}{7}=\frac{35}{7}$$
$$t=5$$

Of course, to answer the final question I need to also find the value of n, so I'll plug $t=5$ into either of the original equations. I'll just use the first equation, because why not?

$$\begin{aligned} 3n+4t&=29 \\ 3n+4(5)&=29 \\ 3n+20&=29 \\ -20\ &-20 \\ 3n&=9 \\ \frac{3n}{3}&=\frac{9}{3} \\ n&=3 \end{aligned}$$

So now we know $n=3$.

And now to answer the final question, which is $n+4t$, by simply plugging in our new-found values:

$$\begin{aligned} &n+4t \\ &=(3)+4(5) \\ &=3+20 \\ &=23 \end{aligned}$$

My final answer is 23, or Choice D.

4. **A.** This is a fairly simple System of Equations. Again, I will choose to use the Elimination Method, since I know that Isolating a single variable will require some ugly fractions when I start dividing. I'd rather avoid this, so instead I'll use Elimination.

First, I'll rewrite the equations so that the x and y terms line up on top and bottom:

$$2x+5y=-23$$
$$-4x+3y=7$$

Now I'll multiply just the top equation by 2:

$$(2)(2x+5y=-23)$$
$$-4x+3y=7$$

$$4x+10y=-46$$
$$-4x+3y=7$$

By doing so, you can see that I've gotten the same number of x terms in both the top and bottom equations. Now it's simple to add the equations to Eliminate the x terms:

$$\begin{aligned} 4x+10y&=-46 \\ +(-4x+3y&=7) \\ 0x+13y&=-39 \end{aligned}$$

Now that I've eliminated the x terms, I can solve for y:

$$13y=-39$$
$$\frac{13y}{13}=\frac{-39}{13}$$
$$y=-3$$

I know that the y-value or *y*-coordinate of the solution will be $y=-3$. That leaves me with only one possible answer choice, which is Choice A.

I could stop here if I'm confident in my work, or I can check my work by plugging $y=-3$ into either of the original equations. If I get $x=-4$ as a result, then I've proven my answer true. You can check for yourself if you want!

5. **D.** Here's our first real Word Problem with Systems of Equations in the practice set. We'll need to set up two equations before we can solve.

First, let's make an equation for the total number of items Ian purchased, which is 9. Add the controllers (I'll use the variable c) and video games (I'll use the variable v):

$$c + v = 9$$

Now let's turn our attention to the amount of money Ian spent on his purchase, a total of \$250. Each video game costs \$30, so $30v$ would represent the total amount Ian spent on games. Each controller costs \$25, so $25c$ is the total amount he spent on controllers. Add them together to get the total cost of \$250:

$$30v + 25c = 250$$

Now we have our System ready to solve: two variables, and two unique equations. Let's put them together:

$$\begin{aligned} c + v &= 9 \\ 30v + 25c &= 250 \end{aligned}$$

I think using the "ISS Method" would be ideal here, because it's easy to Isolate one of the variables in the top equation. I'll isolate *c*:

$$\begin{aligned} c + v &= 9 \\ -v \quad & -v \\ c &= 9 - v \end{aligned}$$

Now I can Substitute in for c in the second equation:

$$\begin{aligned} 30v + 25c &= 250 \\ 30v + 25(9 - v) &= 250 \end{aligned}$$

And then distribute, combine like terms, and solve for the value of v using **Basic Algebra 1**:

$$\begin{aligned} 30v + 25(9 - v) &= 250 \\ 30v + 225 - 25v &= 250 \\ 5v + 225 &= 250 \\ -225 \quad & -225 \\ 5v &= 25 \\ \frac{5v}{5} &= \frac{25}{5} \\ v &= 5 \end{aligned}$$

Now we know that Ian purchased 5 video games. We can return to our original equations and plug $v = 5$ into either of them to get the number of controllers he bought:

$$\begin{aligned} c + v &= 9 \\ c + (5) &= 9 \\ -5 \quad & -5 \\ c &= 4 \end{aligned}$$

And there we go! Ian bought 4 controllers, or Choice D.

6. **D.** Here's our first System of Inequalities for the practice set so far. Notice that the bottom inequality is simpler than the top one, because the bottom one only has a single variable, x. So let's start with the bottom one and Isolate x:

$$\begin{aligned} \frac{4x}{4} &> \frac{12}{4} \\ x &> 3 \end{aligned}$$

This tells us that the *lowest* value x can have is "3" (technically, x must be *greater* than 3). Notice also that the top inequality is focused on the *lowest* value y can have, which is based on the value of x.

To get the lowest-possible y value, we should also use the lowest-possible x value, which we now know is 3.

Plug in 3 for x:

$$\begin{aligned} y &> 4x + 4 \\ y &> 4(3) + 4 \\ y &> 12 + 4 \\ y &> 16 \end{aligned}$$

And so, we find our final answer, that y can be any value higher than 16, or Choice D.

7. **8.** Although I don't enjoy working with ugly fractions in my Systems of Equations, this question really leaves us no choice. But I do have a workaround - why not just get rid of the fractions in both equations? This is easily done if I multiply the entire top equation by "2" and the bottom equation by "4", which gives:

$$\begin{aligned} 3y + 1x &= 4 \\ 5y + 7x &= -36 \end{aligned}$$

Now I can easily Isolate x in the top equation and use the "ISS Method":

$$\begin{aligned} 3y + 1x &= 4 \\ -3y \quad &\quad -3y \\ x &= 4 - 3y \end{aligned}$$

Now it's easy to Substitute in for x in the bottom equation (referring to the equations *after* I got rid of the fractions):

$$\begin{aligned} 5y + 7x &= -36 \\ 5y + 7(4 - 3y) &= -36 \end{aligned}$$

Time to distribute and combine like terms:

$$\begin{aligned} 5y + 7(4 - 3y) &= -36 \\ 5y + 28 - 21y &= -36 \\ -16y + 28 &= -36 \end{aligned}$$

Now I can solve for y:

$$\begin{aligned} -16y + 28 &= -36 \\ -28 \quad &\quad -28 \\ -16y &= -64 \\ \frac{-16y}{-16} &= \frac{-64}{-16} \\ y &= 4 \end{aligned}$$

Great, now I know that $y = 4$. Unfortunately, I've yet again solved for the wrong variable (I seem to have a knack for this) since the question asks for the value of y (or is it possible that the test itself, and these practice questions, have been *purposefully designed* to encourage you to solve for the wrong variable? Hint, hint…)

Either way, it's very easy to finish the question by plugging in $y = 4$ into any of the original equations.

I'd still rather avoid fractions, so I'll use one of my "de-fractioned" versions of the equations:

$$\begin{aligned} 3y + 1x &= 4 \\ 3(4) + x &= 4 \\ 12 + x &= 4 \\ -12 \quad &\quad -12 \\ x &= -8 \end{aligned}$$

Don't forget, the question asks for $-x$, which is $-(-8)$ for a final value of 8!

8. **9.** This is a basic Systems of Equations that's easy to solve either with the "ISS Method" or the Elimination Method. I'll use "ISS" since it's what I normally choose first. Let's Isolate the x in the top equation:

$$\begin{aligned} x + 4y &= 1 \\ -4y \quad &\quad -4y \\ x &= 1 - 4y \end{aligned}$$

Now Substitute this in for x in the second equation:

$$\begin{aligned} 2x + 10y &= -2 \\ 2(1 - 4y) + 10y &= -2 \end{aligned}$$

Next, distribute and combine like terms:

$$\begin{aligned} 2(1 - 4y) + 10y &= -2 \\ 2 - 8y + 10y &= -2 \\ 2 + 2y &= -2 \end{aligned}$$

And now solve for y using **Basic Algebra 1**:

$$\begin{aligned} 2 + 2y &= -2 \\ -2 \quad &\quad -2 \\ 2y &= -4 \\ \frac{2y}{2} &= \frac{-4}{2} \\ y &= -2 \end{aligned}$$

Well, what do you know. Yet again, I've found one variable, but the question asks for the other one (are you noticing a pattern yet?)

Anyway, it's easy enough to fix. Just plug $y = -2$ into either of the original equations:

$$x + 4y = 1$$
$$x + 4(-2) = 1$$
$$x - 8 = 1$$
$$+8 \quad +8$$
$$x = 9$$

And voila, there we go: the value of x is 9.

9. **B.** This Systems of Equations question gives us some ugly decimals to work with. I prefer to avoid decimals and fractions when I use the "ISS Method." Therefore, this question seems like a good candidate for the Elimination Method.

Notice that I can multiply the whole bottom equation by 3, which will set up my y terms to cancel out:

$$1.6x + 2.4y = 2.8$$
$$(3)(3.5x + .8y = -.55)$$

$$1.6x + 2.4y = 2.8$$
$$10.5x + 2.4y = -1.65$$

Now I can subtract the bottom equation from the top equation and cancel the y terms, then solve for x. I'll want to use my calculator a little bit since there are so many ugly decimals to work with:

$$1.6x + 2.4y = 2.8$$
$$-(10.5x + 2.4y = -1.65)$$
$$-8.9x = 4.45$$
$$\frac{-8.9x}{-8.9} = \frac{4.45}{-8.9}$$
$$x = -.5$$

So, despite all of the ugly decimal values, by using the Elimination Method, plus our calculator, and trusting the Algebra, we've been able to find a clean value for the x-coordinate of the intercept point of the two equations, at $x = -.5$ or Choice B.

10. **B.** This is an easy problem, but there's a small twist: we have *two* variables, but only *one* equation (remember that $\frac{6y}{3x}$ doesn't have a balanced equation with an equal sign, so it's an "expression," not an "equation.")

Instead of solving for an *individual* variable like x or y, we're solving for a relationship of *two* variables $\frac{x}{y}$. So, this is what I call a "Combo Variable" question, as described in the lesson.

Luckily, the "ISS Method" will work just fine. First, let's isolate a variable from the equation:

$$\frac{x}{2y} = 1$$
$$(2y)\frac{x}{2y} = 1(2y)$$
$$x = 2y$$

That was easy! Now Substitute $x = 2y$ into the expression:

$$\frac{6y}{3x}$$
$$= \frac{6y}{3(2y)}$$

And now clean up:

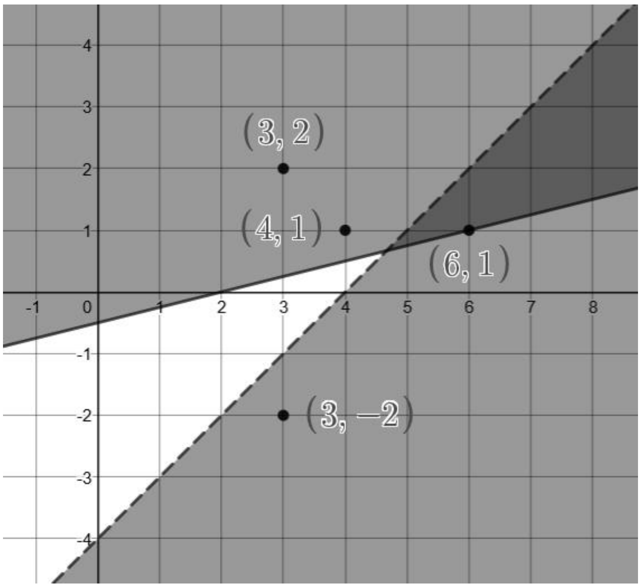
$$\frac{6y}{3(2y)}$$
$$= \frac{6y}{6y}$$
$$= 1$$

And there we have our final value of 1 or Choice B.

11. **D.** In this question, we face another System of Inequalities. There are several ways to solve this problem. Most students will find that the two best ways are either 1) sketching graphs of the two inequalities and using a visual approach or 2) testing each of the Answer Choices by plugging them into the given inequalities.

In my opinion, you should do whichever method feels more comfortable. The graphing method will probably be slightly faster overall, but not by much.

Below I've included a graph of the two inequalities and the four Answer Choice coordinates. You should be able to produce a similar graph using the lessons we learned in **Linear Equations**:



Notice that only point $(6,1)$ is within the double-shaded region (technically, it sits just on the boundary of the acceptable region, but it's on a solid line that is inclusive of any points equal to that line).

Alternately, you could just test each of the four answer choices by plugging the x- and y-values into both equations. It will take a bit of time, but you will find that Choices A, B, and C produce false inequalities, and only Choice D produces a pair of true inequalities. Either way, only Choice D provides a point that satisfies both inequalities at the same time.

12. **2750.** This is one of the easiest systems in this practice section! Notice that the question asks for $7x+7y$. We can simply add the two equations to each other directly. Look what happens:

$$\begin{aligned} 4x+2y&=850 \\ +(3x+5y&=1900) \\ 7x+7y&=2750 \end{aligned}$$

And we're done! We see that $7x+7y=2{,}750$. No need to use "ISS" or Elimination. Just notice what the question asks for and realize that you're already almost there, then add the equations together.

13. **A**. This is one of those Word Problems that requires us to *set up* a System of Inequalities, but we don't actually have to *solve* it.

It can feel like "information overload" with such a long word problem, so just pick one element at a time to focus on. For example, we know that Thomas can't work for more than 50 hours per week. In other words, the sum total of x and y must be less than or equal to 50 hours:

$$x+y\le 50$$

This setup already eliminates Choices C and D, which have the inequality sign facing the wrong direction.

Now move onto the financial side. Thomas needs to earn at least $900 per week. His x hours of clay figurines earn him $15 per hour, so his total income from the figurine business will be $15x$. His y hours of lawn mowing earn him $20 per hour, so his total income from lawn mowing will be $20y$.

Add his income up and make sure that it's more than $900:

$$15x+20y\ge 900$$

This eliminates the remaining Choice B, which also has the inequality sign facing the wrong direction, and we can see that Choice A has the two inequalities we need.

14. **A.** This question barely qualifies as a "System of Equations," since we're only given *one* equation to start with. However, we do have the characteristic two-variable algebra question with x and y, and the answer choices provide a second possible equation.

So, I'd tend to think of this as a "Combo Variable"-type question, since all of our answer choices are based on *relationships* of two variables, rather than the value of individual variables.

I'm not quite sure what to do first, so you know what? I'm just going to cross-multiply the original equation. This is usually not a bad idea - when you *can* cross-multiply, you usually *should* - so I'll try it and see what I think of next:

$$\frac{x}{y-2x}=\frac{2}{13}$$
$$(13)(x)=(2)(y-2x)$$

OK, now let's distribute, combine like terms, and generally clean this equation up:

$$(13)(x)=(2)(y-2x)$$
$$13x=2y-4x$$
$$+4x \qquad +4x$$
$$17x=2y$$

Alright, now I have an idea: why don't I use Algebra to make my current equation look more like the answer choices?

I'll shoot for $\frac{x}{y}$ because it's the simplest option available in the answer choices:

$$17x=2y$$
$$\frac{17x}{y}=\frac{2y}{y}$$
$$\frac{17x}{y}=2$$
$$17(\frac{x}{y})=2$$
$$(\frac{1}{17})17(\frac{x}{y})=2(\frac{1}{17})$$
$$\frac{x}{y}=\frac{2}{17}$$

And check it out - by experimenting and using the answer choices for inspiration, I've found an equation that looks exactly like Choice A.

15. **710.** This is a Word Problem that needs to be set up into a System of Equations first. Let's use the variable s to represent the calories in a steak, and p to represent the calories in an order of mashed potatoes.

We know that a steak has 400 more calories than an order of mashed potatoes. In other words, take mashed potato calories and add $+400$, and that will give the calories in a steak:

$$s=p+400$$

We also have the total calories (of 2750) for a mixed order of 3 steaks and 2 mashed potatoes:

$$3s+2p=2750$$

Here's our completed System setup:

$$s=p+400$$
$$3s+2p=2750$$

At this point, it would be easy to use the "ISS Method." We already have s Isolated, so let's move straight onto Substitution into the second equation:

$$3s+2p=2750$$
$$3(p+400)+2p=2750$$

Now we can distribute, combine terms, and solve for p:

$$3(p+400)+2p=2750$$
$$3p+1200+2p=2750$$
$$5p+1200=2750$$
$$-1200 \quad -1200$$
$$5p=1550$$
$$\frac{5p}{5}=\frac{1550}{5}$$
$$p=310$$

Now we know that $p=310$; there are 310 calories in an order of mashed potatoes. But don't forget that we're supposed to get the calories in a *steak*, not in potatoes. Plug $p=310$ back into either of the original equations we set up to solve for s:

$$s=p+400$$
$$s=(310)+400$$
$$s=710$$

So, a steak must contain 710 calories.

16. **28.** This question is a callback to **Linear Equations**. If two equations represent the *same* line, then the equations of the two lines must be exactly equal to each other.

So, let's try to make these two equations equal. The easiest way would just be to multiply the entire first equation by 4:

$$(4)(2x+y=7)$$
$$8x+4y=28$$

Now compare this new equation to the second given equation:

$$8x+4y=28$$
$$8x+4y=p$$

And it's clear to see that, if the two equations are identical, then the value of p must be 28.

17. **8.** This is a basic Systems of Equations question without a word problem. We can use the "ISS Method" easily, because the first equation has already Isolated the n. Now just Substitute it into the second equation:

$$2n = \sqrt{6x}$$
$$2(2\sqrt{3}) = \sqrt{6x}$$

Now distribute and Solve for x. Better be solid on your **Exponents & Roots** Algebra!

$$2(2\sqrt{3}) = \sqrt{6x}$$
$$4\sqrt{3} = \sqrt{6x}$$
$$(4\sqrt{3})^2 = (\sqrt{6x})^2$$
$$16(3) = 6x$$
$$48 = 6x$$
$$\frac{48}{6} = \frac{6x}{6}$$
$$8 = x$$

The value of x must be 8**.**

Although the Algebra above is fairly straightforward, there is still room for **Careless Mistakes**, especially when squaring both sides. Be careful!

18. **.5.** This is a straightforward System of Equations. We can immediately use the Elimination Method and add the bottom equation to the top equation, which will instantly cancel the x terms:

$$x + y = 2$$
$$+(-x + 4y = .5)$$
$$0x + 5y = 2.5$$

Now it's easy to solve for y using **Basic Algebra 1**:

$$\frac{5y}{5} = \frac{2.5}{5}$$
$$y = .5$$

And we're done. The value of y is $.5$.

19. **D.** This System of Equations is just begging for a simple Elimination Method. The y terms on top and bottom are already set up to cancel out if we just subtract the bottom equation from the top equation. Watch your negative signs carefully!

$$11x - 15y = 212$$
$$-(10x - 15y = 205)$$
$$1x + 0y = 7$$
$$x = 7$$

Great, we've already found that $x = 7$. Of course, to find the final answer to the question $x - y$, we'll also need the value of y.

Plug $x = 7$ back into either of the original equations:

$$10x - 15y = 205$$
$$10(7) - 15y = 205$$

Now solve for the value of y:

$$10(7) - 15y = 205$$
$$70 - 15y = 205$$
$$-70 \qquad -70$$
$$-15y = 135$$
$$\frac{-15y}{-15} = \frac{135}{-15}$$
$$y = -9$$

And to finish off the question, let's take care of $x - y$:

$$x - y$$
$$= (7) - (-9)$$
$$= 7 + 9$$
$$= 16$$

And there we go - the final answer is 16.

20. **C.** This System of Equations is a callback to the Discriminant from the lesson on **The Quadratic Formula**. Remember that the Discriminant is used to find *how many* solutions exist to a Quadratic Equation of form $ax^2 + bx + c = 0$.

Before we can use the Discriminant, we need to combine the two equations and then set the result equal to 0.

We'll use the "ISS" method to combine the equations. It's easiest to Isolate y from the bottom equation, which is a simpler equation overall:

$$\begin{aligned} y - 4x + 3.5 &= 0 \\ +4x \qquad &+4x \\ y + 3.5 &= 4x \\ -3.5 \qquad &-3.5 \\ y &= 4x - 3.5 \end{aligned}$$

Now we can Substitute this in for y in the top given equation:

$$\begin{aligned} y &= 2x^2 - 2x + 1 \\ 4x - 3.5 &= 2x^2 - 2x + 1 \end{aligned}$$

Now we need to set this equal to 0 in order to use the Discriminant:

$$\begin{aligned} y &= 2x^2 - 2x + 1 \\ 4x - 3.5 &= 2x^2 - 2x + 1 \\ -4x \qquad &\qquad -4x \\ -3.5 &= 2x^2 - 6x + 1 \\ +3.5 \qquad &\qquad +3.5 \\ 0 &= 2x^2 - 6x + 4.5 \end{aligned}$$

OK, now we'll use the Discriminant. I'm not going to do a full review because you can find all the relevant info in the lesson on **The Quadratic Formula**. The Discriminant is:

$$b^2 - 4ac$$

Now plug in the appropriate values from our Quadratic Equation that we've set up:

$$\begin{aligned} &b^2 - 4ac \\ &= (-6)^2 - 4(2)(4.5) \end{aligned}$$

And evaluate:

$$\begin{aligned} &(-6)^2 - 4(2)(4.5) \\ &= 36 - 36 \\ &= 0 \end{aligned}$$

The value of the Discriminant for this equation is "0", which means there is exactly *one* solution to the System of Equations, or Choice C. Again, all of this is explained in the lesson on **The Quadratic Formula**, so review if necessary.

21. **C.** This is a basic System of Equations that is probably best solved with the Elimination Method, since the "ISS Method" will force us to use a bunch of ugly fractions. (Remember, the "ISS Method" will *always* work, but on test day we're looking for the easiest and fastest path to a solution).

So, let's multiply the top equation by 5 and the bottom equation by 4, which will allow us to cancel some terms with Elimination:

$$\begin{aligned} &(5)(4x - 5y = 8) \\ &(4)(5x - 4y = 28) \end{aligned}$$

$$\begin{aligned} 20x - 25y &= 40 \\ 20x - 16y &= 112 \end{aligned}$$

Now we have a perfect Elimination setup. Subtract the bottom equation from the top equation to Eliminate the x terms. Be sure to watch your negative signs carefully:

$$\begin{aligned} 20x - 25y &= 40 \\ -(20x - 16y &= 112) \\ 0x - 9y &= -72 \\ -9y &= -72 \end{aligned}$$

Now we can solve for the value of y:

$$\begin{aligned} \frac{-9y}{-9} &= \frac{-72}{-9} \\ y &= 8 \end{aligned}$$

OK, so now we know that $y = 8$.

We still need the value of x before we can answer the final question $x + y$. So, plug $y = 8$ into either of the original equations:

$$\begin{aligned} 4x - 5y &= 8 \\ 4x - 5(8) &= 8 \\ 4x - 40 &= 8 \\ +40 \ &+40 \\ 4x &= 48 \\ \frac{4x}{4} &= \frac{48}{4} \\ x &= 12 \end{aligned}$$

Now we know that $x = 12$. Time to finish the question $x + y$:

$$\begin{aligned} &x + y \\ &= (12) + (8) \\ &= 20 \end{aligned}$$

The final answer is 20, or Choice C.

22. **120.** In this System of Inequalities, it's important to notice that *both* inequalities limit the maximum value of y. We're being asked for the maximum possible value of b, which is given as a *y*-coordinate of point (a, b). In other words, we're trying to find the maximum possible value of y that satisfies both inequalities.

That's why it's so important to realize that *both* inequalities limit the maximum value of y. If we're trying to find the highest-possible y value, we should use the maximum possibility for y in both inequalities.

In other words, we can act like the System really says "equal" instead of "*less than* or equal". We're not interested in the "less than" part, because we only want to *maximize* the value of y:

$$\begin{aligned} y &= -16x + 600 \\ y &= 4x \end{aligned}$$

We can easily solve our new System of Equations (not Inequalities anymore) by using the "ISS Method." We've already isolated y in the bottom equation, so go ahead and Substitute into the top equation:

$$\begin{aligned} y &= -16x + 600 \\ 4x &= -16x + 600 \end{aligned}$$

And now solve for the value of x:

$$\begin{aligned} 4x &= -16x + 600 \\ +16x &\quad\ +16x \\ 20x &= 600 \\ \frac{20x}{20} &= \frac{600}{20} \\ x &= 30 \end{aligned}$$

So we know $x = 30$. Now take x and plug it back into either of the original inequalities:

$$\begin{aligned} &y \le 4x \\ &y \le 4(30) \\ &y \le 120 \end{aligned}$$

And we find that y can equal any value less than or equal to 120, making 120 the *maximum* possible value for y (or b, which is a *y*-coordinate).

23. **C.** This is one of those long word problems that asks us to set up a System, but not to actually finish solving it. As always, push through the feeling of "information overwhelm" that most of us feel, and find a single element to focus on first.

We could start with the minimum hires that Christian needs to make. He needs at least 3 art designers, so $x \ge 3$, and he needs 2 copywriters, so $y \ge 2$. These inequalities eliminate Choices A and B right away.

Now all that's left is to focus on the *budget* element of the word problem. The maximum cost of the new hires is $8,800 per week. The cost of each art designer is $850 per week, so $850x$ represents the total weekly cost for art designers. The cost of each copywriter is $900 per week, so $900y$ represents the total weekly cost for copywriters.

Add the costs of art designers and copywriters and make sure it's less than (or equal to) the maximum budget of $8,800:

$$850x + 900y \le 8{,}800$$

And this eliminates Choice D. We're left with only Choice C, which has all of its inequality signs facing in the proper directions.

Notice that the "at least 8 staff members" info didn't even end up mattering, since all four choices give $x + y \ge 8$.

24. **B.** This is a great question that tests our understanding of graphs of Systems! Remember, if you graph a System of Equations, the solutions will be found at the intersection points of the lines or curves. However, this System has *three* equations - the question tells us so, and we can also see *two* linear equations *and* a parabola. That means only points where *all three* lines or curves intersect will count as solutions to the System.

If you look closely, there's only one point where all three lines or curves meet simultaneously. It's at the coordinate $(-1,4)$ if you don't see it. There is only one solution to this System of three equations, so the answer is Choice B.

Many students mistakenly believe that *any* intersection points will count as solutions, and might incorrectly think there are three solutions. Again, a system is only true when *all* the equations in the system are equal. The graph of a system only has solutions where *all* of the lines or curves intersect at the same point.

25. **B.** This is a word problem that needs to be set up into a System of Equations and then solved.

First, we'll need some new variables to represent the number of 2-person carriages (I'll use x) and 3-person carriages (I'll use y). When you're making up new variables, be sure to write down what each variable represents.

So, we know there were a total of 39 carriages:

$$x+y=39$$

And now, let's focus on the number of people. There are 94 total people. Each x carriage holds 2 of them, and each y carriage holds 3 of them:

$$2x+3y=94$$

Now we have our complete System, with one equation for the number of carriages, and a second equation for the number of people:

$$\begin{aligned} x+y&=39 \\ 2x+3y&=94 \end{aligned}$$

I'll solve this using the "ISS Method" since it won't take long to Isolate a variable from the top equation:

$$\begin{aligned} x+y&=39 \\ -y\quad&\quad -y \\ x&=39-y \end{aligned}$$

Now we can Substitute x into our second equation:

$$\begin{aligned} 2x+3y&=94 \\ 2(39-y)+3y&=94 \end{aligned}$$

Now distribute, combine like terms, and solve for y:

$$\begin{aligned} 2(39-y)+3y&=94 \\ 78-2y+3y&=94 \\ 78+y&=94 \\ -78\quad&\quad -78 \\ y&=16 \end{aligned}$$

So, the value of y is 16. It's very helpful to have a written note to help you remember which variable is which - we were using y to represent the *three*-person carriages. And that's what the question asked for, so we're done! The answer is 16, or Choice B.

26. **7.5** This is a basic System of Equations that is probably best solved with the Elimination Method, so that we can avoid the ugly fractions that will arise if we use the "ISS Method".

If we just multiply the top equation by 3, we can easily cancel out the x terms in the next step.

$$\begin{aligned} (3)(-2x+3y&=-12) \\ 6x+2y&=47 \end{aligned}$$

$$\begin{aligned} -6x+9y&=-36 \\ 6x+2y&=47 \end{aligned}$$

Now just add the two equations to each other to cancel the x terms:

$$\begin{aligned} -6x+9y&=-36 \\ +(6x+2y&=47) \\ 0x+11y&=11 \\ 11y&=11 \\ \frac{11y}{11}&=\frac{11}{11} \\ y&=1 \end{aligned}$$

And now we know the Systems have a solution at $y=1$.

We just need to plug $y = 1$ back into either of the original equations to find the value of x:

$$\begin{aligned} 6x + 2y &= 47 \\ 6x + 2(1) &= 47 \\ 6x + 2 &= 47 \\ -2 \quad &\; -2 \\ 6x &= 45 \\ \frac{6x}{6} &= \frac{45}{6} \\ x &= 7.5 \end{aligned}$$

And our final value is $x = 7.5$.

27. **A.** This question gives the coordinates $(0,0)$ as a solution to the system. We should plug in $x = 0$ and $y = 0$:

$$y > 2x + m$$
$$y > -3x - n$$

$$0 > 2(0) + m$$
$$0 > -3(0) - n$$

Then clean up:

$$0 > 2(0) + m$$
$$0 > -3(0) - n$$

$$0 > 0 + m$$
$$0 > 0 - n$$

$$0 > m$$
$$0 > -n$$

OK, one more thing: I'm going to multiply the bottom inequality by -1 to get rid of the negative sign on the n. Remember that the direction of the inequality sign will flip whenever we divide or multiply by a negative:

$$0 > m$$
$$(-1)0 > -n(-1)$$

$$0 > m$$
$$0 < n$$

Now, this final form of the system tells us some very interesting information. The value of m must be less than 0; in other words, m must be a negative number. The value of n must be more than 0; in other words, n must be a positive number.

If there's one thing we can be sure of, it's that *all* positive numbers are greater than *all* negative numbers. It's quite safe to say that m, a negative number, must be less than n, a positive number: $m < n$. That gives us Choice A.

28. **A.** This is a word problem that needs to be set up into a System of Equations. I'll use the variable d to represent drinks sold, and c to represent candy bars sold. We know the sum total of these was 87 for the day:

$$d + c = 87$$

Now let's focus on the total cash involved. The total sales were \$235.50. Each drink is \$3.50, so $3.5d$ covers the total drink sales. Each candy bar is \$2, so $2c$ covers the total candy bar sales. Our setup for total sales is:

$$3.5d + 2c = 235.5$$

Now consider our complete system setup:

$$d + c = 87$$
$$3.5d + 2c = 235.5$$

This is a great chance to use the "ISS Method." Isolate a variable from the top equation:

$$\begin{aligned} d + c &= 87 \\ -c \quad &\; -c \\ d &= 87 - c \end{aligned}$$

Now we can Substitute in for d in our second equation:

$$3.5d + 2c = 235.5$$
$$3.5(87 - c) + 2c = 235.5$$

Start distributing & cleaning up, then solve for c:

$$\begin{aligned} 3.5(87 - c) + 2c &= 235.5 \\ 304.5 - 3.5c + 2c &= 235.5 \\ 304.5 - 1.5c &= 235.5 \\ -304.5 \qquad &\; -304.5 \\ -1.5c &= -69 \\ \frac{-1.5c}{-1.5} &= \frac{-69}{-1.5} \\ c &= 46 \end{aligned}$$

Now we know that the snack stand sold 46 candy bars. But remember, we're solving for *drinks*, not candy bars. Go ahead and plug $c = 46$ back into one of our original equations:

$$\begin{aligned} d + c &= 87 \\ d + (46) &= 87 \\ -46 \quad &-46 \\ d &= 41 \end{aligned}$$

And now we know that the snack stand sold 41 drinks, or Choice A.

29. **A.** This is a basic System of Equations question, although it's the first one of these questions that involves Algebra 2, instead of Algebra 1. That doesn't change how we deal with Systems, though.

This System is in great shape for the "ISS Method." The y is already isolated in the first equation, so let's Substitute in directly to the bottom equation. Here's what we get:

$$x(x-2) - 3x = 14$$

Now it's time to distribute and combine like terms.

$$\begin{aligned} x(x-2) - 3x &= 14 \\ x^2 - 2x - 3x &= 14 \\ x^2 - 5x &= 14 \end{aligned}$$

This has turned into a Quadratic Equation. As we studied in **Basic Algebra 2**, we should set it equal to 0:

$$\begin{aligned} x^2 - 5x &= 14 \\ -14 \quad &-14 \\ x^2 - 5x - 14 &= 0 \end{aligned}$$

Now, let's see if we can Factor this:

$$\begin{aligned} x^2 - 5x - 14 &= 0 \\ (x-7)(x+2) &= 0 \end{aligned}$$

I'm not going over the basic methods of Factoring, since we've already covered that in **Basic Algebra 2**.

Now we can easily read off our solutions to the equation, which are $x = 7$ and $x = -2$. The question states that $x < 0$, the solution we want is $x = -2$, or Choice A.

30. **D.** The word problem tells us we're looking for a time when the prices of A and B are *equal* to each other. Therefore, we can just set the two price equations equal to each other:

$$\begin{aligned} a &= b \\ 450 - 25w &= 330 - 5w \end{aligned}$$

Now we've got a basic one-variable Algebra 1 question that's not hard to solve:

$$\begin{aligned} 450 - 25w &= 330 - 5w \\ +25w \quad &\quad +25w \\ 450 &= 330 + 20w \\ -330 \quad &-330 \\ 120 &= 20w \\ \frac{120}{20} &= \frac{20w}{20} \\ 6 &= w \end{aligned}$$

So, we've found that $w = 6$. Is that our final answer? Only if you've completely lost track of what we're solving for! Our final answer is supposed to be a *dollar* value, not a number of weeks.

To finish the question, we have to plug $w = 6$ into either of the pricing equations. Either equation will work, since we're asked when the prices are *equal to* each other:

$$\begin{aligned} b &= 330 - 5w \\ b &= 330 - 5(6) \\ b &= 330 - 30 \\ b &= 300 \end{aligned}$$

There we go! After 6 weeks, the two processors both cost \$300, or Choice D**.**

31. **C.** This question is a callback to **Linear Equations**. Both of these equations are Linear Equations. If two Linear Equations have "no solutions," that means the lines never intersect, and the two lines must be parallel - so they have the same slope.

A solid way to solve these questions is to put both equations into $y = mx + b$ form. I'll assume that you can do that on your own at this point. Here's what we get:

$$y = -\frac{2}{3}x + \frac{10}{3}$$
$$y = -\frac{a}{4}x + \frac{1}{2}$$

Now compare the slope or m values of the two equations. In fact, we should set them equal to each other, because we know the lines must be parallel:

$$-\frac{2}{3} = -\frac{a}{4}$$

Now solve for the value of a:

$$(-1) - \frac{2}{3} = -\frac{a}{4}(-1)$$
$$\frac{2}{3} = \frac{a}{4}$$
$$(4)(2) = (3)(a)$$
$$8 = 3a$$
$$\frac{8}{3} = \frac{3a}{3}$$
$$\frac{8}{3} = a$$

So the value of a that makes the two lines parallel (and therefore "no solutions") is $\frac{8}{3}$, and therefore Choice C.

32. **A.** As in Question 20, this System of Equations is a callback to the Discriminant from the lesson on **The Quadratic Formula**. Remember that the Discriminant is used to find *how many* solutions exist to a Quadratic Equation of form $ax^2 + bx + c = 0$.

Before we can use the Discriminant, we'll need to combine the two given equations and then set the result equal to 0.

We'll use the "ISS" method to combine the equations. The variable y is already isolated in the bottom equation. Now we can Substitute this in for y in the top equation:

$$x = 4[(x+2)(x+1)] + 32$$

We'll need to FOIL first, as we learned in **Basic Algebra 2**:

$$x = 4[x^2 + 3x + 2] + 32$$

Then distribute and combine like terms:

$$x = 4x^2 + 12x + 8 + 32$$
$$x = 4x^2 + 12x + 40$$

Now we need to set this equal to 0 in order to use the Discriminant:

$$x = 4x^2 + 12x + 40$$
$$-x \qquad\qquad -x$$
$$0 = 4x^2 + 11x + 40$$

OK, now we'll use the Discriminant. I'm not going to do a full review because you can find all the relevant info in the lesson on **The Quadratic Formula**. The Discriminant is:

$$b^2 - 4ac$$

Now plug in the appropriate values from our Quadratic Equation that we've set up:

$$b^2 - 4ac$$
$$= (11)^2 - 4(4)(40)$$

And evaluate:

$$(11)^2 - 4(4)(40)$$
$$= 121 - 640$$
$$= -519$$

The value of the Discriminant for this equation is *negative*, which means there are *no solutions* to the System of Equations, or Choice A.

Again, all of this is explained in the lesson on **The Quadratic Formula**, so review if necessary.

33. **D.** This is another word problem where we simply need to set up a System, but don't actually need to finish solving it. As usual with these problems, the question attempts to use "information overwhelm" to intimidate us into giving up before we start. And, as always, we can beat them at their own game by focusing on one single element of the question to work with first.

For example, we know that a total of 60 boxes were ordered. That's easy: it just means the sum total of s standard boxes and e elite boxes must be 60:

$$s+e=60$$

This allows us to quickly eliminate Choice A. We can't eliminate anything else, because you could rewrite our equation above as $s=60-e$ or $e=60-s$.

Now let's focus on the element of size. The total order has a volume of 2,100 cubic inches. Each standard box has a volume of 30 cubic inches, so $30s$ represents the *total* volume of all the standard boxes. Similarly, the expression $45e$ would represent the total volume of all the *elite* boxes. Add the total volumes of the two types of boxes together and you get:

$$30s+45e=2100$$

Now we can eliminate our remaining choices - for example Choice B has the e and s reversed - except for Choice D, which has the correct equations for the System. Like most of these "Words into Systems" questions, they look *much* harder than they actually are.

34. $\frac{1}{2}$ or **.5.** Just like Question 31, this question is a callback to **Linear Equations**. Both of these equations are Linear Equations. If two Linear Equations have "infinite solutions," that means that they overlap at infinite points, and the two "different" lines are actually *the same line*.

A solid way to solve these questions is to put both equations into $y=mx+b$ form. I'll take for granted that you can do that on your own at this point. You'll have to be solid with your **Fractions**. Here's what we get:

$$y=2x-\frac{64}{3}$$
$$y=\frac{a}{b}x-\frac{6}{b}$$

Now, we're also solving for the "Combo Variable" of $\frac{b}{a}$ in this question.

Can I point something out? Take another look at our two equations:

$$y=2x-\frac{64}{3}$$
$$y=\frac{a}{b}x-\frac{6}{b}$$

The bottom equation has a slope of $\frac{a}{b}$, and the top equation has a slope of 2. We know these two equations represent the exact same line, so the slopes are the same. That means $\frac{a}{b}=2$.

And, the question is asking for the value of $\frac{b}{a}$, the *reciprocal* of the value we already have. All we have to do is flip $\frac{a}{b}=2$ upside-down:

$$\frac{a}{b}=\frac{2}{1}$$
$$\frac{b}{a}=\frac{1}{2}$$

And we get our final answer: $\frac{b}{a}$ is equal to $\frac{1}{2}$ or $.5$.

35. **6.** This is a basic Systems of Equations problem that will also use **Basic Algebra 2** for the solution.

First, this setup is perfect for the "ISS Method." We already have an Isolated y-term in the bottom equation. Let's Substitute it directly in for y in the top equation:

$$3+x=x^2-2x-15$$

Now we have the makings of a Quadratic Equation, so let's set it equal to 0:

$$\begin{aligned} 3+x&=x^2-2x-15 \\ -3\quad &\qquad\qquad -3 \\ x&=x^2-2x-18 \\ -x\quad &\quad -x \\ 0&=x^2-3x-18 \end{aligned}$$

Now let's see if we can Factor this equation:

$$\begin{aligned} 0&=x^2-3x-18 \\ 0&=(x-6)(x+3) \end{aligned}$$

Awesome - it factored easily. Now we can read off the solutions of $x=6$ and $x=-3$. The question states that $x>0$, so our final solution must be 6**.**

If you understand our original setup but not the algebra work that followed it, be sure to review the lesson on **Basic Algebra 2**.

36. **120.** Finally, we have a System from a Word Problem that's actually more difficult than most of the ones we've solved so far.

The setup for this question is all-important. First of all, let's look at the final situation, which we can set up as a Proportion:

$$\frac{5\text{ male}}{8\text{ total}}=\frac{\text{male}}{\text{total}}$$

Now let's back up and start with the beginning:

$$\frac{110\text{ male}}{110\text{ male}+78\text{ female}} =\frac{110\text{ male}}{188\text{ total}}$$

After that, we add another 60 female wolves to the total:

$$\frac{110\text{ male}}{188\text{ total}+60\text{ female}} =\frac{110\text{ male}}{248\text{ total}}$$

Now we also need to add an additional x males. Be sure to add those males to the top *and* the bottom total!

$$\frac{110\text{ male}+x\text{ additional males}}{248\text{ total}+x\text{ additional males}}$$

And now we set this equal to the final proportion of $\frac{5}{8}$:

$$\frac{110+x}{248+x}=\frac{5}{8}$$

This is our completed setup for the problem. Now it's time to solve for x. Start by cross-multiplying:

$$\begin{aligned} \frac{110+x}{248+x}&=\frac{5}{8} \\ (8)(110+x)&=(5)(248+x) \\ 880+8x&=1240+5x \end{aligned}$$

Now keep solving using **Basic Algebra 1**:

$$\begin{aligned} 880+8x&=1240+5x \\ -5x\quad &\qquad\quad -5x \\ 880+3x&=1240 \\ -880\qquad &\quad -880 \\ 3x&=360 \\ \frac{3x}{3}&=\frac{360}{3} \\ x&=120 \end{aligned}$$

And now we know the value of x, the number of additional male wolves, must be 120.

37. **A.** Just like Question 24 & 31, this question is a callback to **Linear Equations**. Both of these equations are Linear Equations. If two Linear Equations have "infinitely many solutions," that means that they overlap at infinite points, and the two "different" lines are actually *the same line*.

A solid way to solve these questions is to put both equations into $y=mx+b$ form. I'll take for granted that you can do that on your own at this point.

Here's what we get once we're finished:

$$y = \frac{5}{8}x - \frac{5}{2}$$
$$y = bx - \frac{5}{2}$$

Notice that the *y*-intercepts of these two equations are already the same: $-\frac{5}{2}$.

Therefore, all that's left is to make sure the two slopes are identical. In other words, $b = \frac{5}{8}$. And we're done - that gives Choice A.

38. **2** or **8.** We're asked for an n that will plug into both functions to make them equal to each other. As you know from **Functions**, the *n*-value is an *input* or a number we plug in for x in both equations.

There are two ways to solve this Algebra 2 graph-based System of Equations. The first (and more time-consuming) way is to set the two functions equal to each other, like this:

$$-x + 9 = -\frac{1}{2}(x-4)^2 + 9$$

Then you can solve this using **Basic Algebra 2** by setting it equal to 0 and Factoring it for the two solutions.

If you go this route, I suggest first multiplying everything by 2 to eliminate the pesky fraction.

I will not show the complete Algebra solution steps here, because by this point you should have mastered Basic Algebra 2 a long time ago (of course, review the previous lesson if you're feeling shaky).

The other option is to simply sketch the Linear Function $g(x) = -x + 9$ onto the existing grid. I prefer this, because the question was polite enough to give us a detailed grid that allows us to mark exact points as we graph our new line. In this case, it's easier and faster to draw a simple line than it is to solve a multi-step Algebra 2 problem.

As you know from **Linear Equations**, this line will start at a *y*-intercept of "$+9$" and descend with a slope of -1, or falling by 1 unit for each unit we move to the right.

The only major warning is that the axes are *not* "square." The *y*-axis is marked in tick marks of "1 unit" but the *x*-axis is marked in tick marks of "$.5$ units"! Be very careful as you plot your line.

Whether you solve by Algebra 2 or by graphing the Linear Equation, you will find solutions or intersect points at $x = 2$ and $x = 8$.

Either 2 or 8 is acceptable as a final answer. Be sure you give an *x*-value as your solution, not a *y*-value! If you're mixed up between whether you're solving for x or for y, I recommend you review the previous lesson on **Functions**.

39. **C.** A relatively difficult Word Problem with a System of Equations setup. Let's use the variables h for cups of Happy Hiker mix and a for cups of Adventure Animal trail mix.

We know that the two mixes added together give a total of one cup:

$$h + a = 1$$

We also know that the total calories in this cup is 315. However, we *don't* know the total calories in *one cup* of each of the trail mixes, because they're given in the question as "awkward" amounts (one-third of a cup of Happy Hiker and two cups of Adventure Animal).

So, set up for the calories in *one* cup of each of the trail mixes. There are 120 calories in one-third cup h of Happy Hiker:

$$\frac{1}{3}h = 120$$

There are 480 calories in two cups of Adventure Animal:

$$2a = 480$$

Note that these are not a *System* of Equations - they are both independent (they don't share any variables between them). I just want to know how many calories are in a cup of each trail mix first. So, let's solve both of them:

$$(3)\frac{1}{3}h = 120(3)$$
$$h = 360$$

So, a single cup of Happy Hiker contains 360 calories.

$$\frac{2a}{2} = \frac{480}{2}$$
$$a = 240$$

And, a single cup of Adventure Animal equals 240 calories.

Now I can work on the total calories in Tara's mixture:

$$360h + 240a = 315$$

This equation will give the total calories of Tara's mixture. Let's look at our completed System setup:

$$h + a = 1$$
$$360h + 240a = 315$$

Now I can solve the system using the "ISS Method". It's easy to Isolate a from the top equation:

$$h + a = 1$$
$$-h \qquad -h$$
$$a = 1 - h$$

And now I can Substitute this for a in the second equation:

$$360h + 240a = 315$$
$$360h + 240(1-h) = 315$$

Now distribute, combine like terms, and Solve for h:

$$360h + 240(1-h) = 315$$
$$360h + 240 - 240h = 315$$
$$120h + 240 = 315$$
$$-240 \quad -240$$
$$120h = 75$$
$$\frac{120h}{120} = \frac{75}{120}$$
$$h = \frac{75}{120}$$

We need to simplify the fraction $h = \frac{75}{120}$:

$$h = \frac{75}{120}$$
$$h = \frac{15}{24}$$
$$h = \frac{5}{8}$$

And now we have our final answer: Tara's custom mixture has $\frac{5}{8}$ of a cup of Happy Hiker trail mix, or Choice C.

40. **C.** This System of Equations makes use of **Exponent** rules. The key is to realize that 16 can be rewritten as 2^4. This is extremely important, because then we have the same base of "2" on the top and bottom of the fraction:

$$\frac{2^x}{(2^4)^y}$$

Notice that the bottom of the fraction has an exponent raised to another exponent. We know this means that those two exponents multiply together:

$$\frac{2^x}{2^{4y}}$$

Then, if there is a division problem with the same base number on top and bottom, we can simplify by subtracting the bottom exponent from the top exponent:

$$\frac{2^x}{2^{4y}} = 2^{x-4y}$$

Before you go any further, notice something very interesting: the question *gives* us the value of $x - 4y$, which happens to be the exact same expression as our exponent! If $x - 4y = 8$, then we can directly substitute that in, and finish the question right away:

$$2^{x-4y}$$
$$= 2^8$$

And this matches Choice C, so we're done!

41. **B.** This question is almost identical to Pretest Question 2. Let's use the variables a, b, and *n* to represent our three unknown numbers. They have a sum of 544:

$$a + b + n = 544$$

We also know that n is 30% less than the sum of *a* and b:

$$n = .7(a+b)$$

If you don't understand why I'm using a multiple of "$.7$" to represent "30% less than," then you should review the previous lesson on **Percents**.

Now we have our System set up and ready:

$$a + b + n = 544$$
$$n = .7(a+b)$$

Except there's one big problem. We have *three* unique variables, but only *two* equations. That means we can't solve for the value of n.

But, what if we treated the sum of $a+b$ as a single variable? Let's call it s. We could substitute *s* for $a+b$ in both of our equations:

$$s+n=544$$
$$n=.7(s)$$

And now we have a System of Equations with only two variables and two equations.

We can use the "ISS Method" because n is already Isolated. Go ahead and Substitute in for n in the top equation:

$$s+n=544$$
$$s+(.7s)=544$$

And now solve for s:

$$s+(.7s)=544$$
$$1.7s=544$$
$$\frac{1.7s}{1.7}=\frac{544}{1.7}$$
$$s=320$$

But, we want the value of n, so now plug $s=320$ back into one of our earlier equations:

$$s+n=544$$
$$(320)+n=544$$

And solve for n:

$$(320)+n=544$$
$$-320 \quad\quad -320$$
$$n=244$$

If $n=244$, this gives us our final answer of Choice B**.**

42. **5.** Just like Question 24, 31 & 37, this question is a callback to **Linear Equations**. Both of the given equations are Linear Equations. If two Linear Equations have "infinitely many solutions," that means that they overlap at infinite points, and the two "different" lines are actually *the same line*.

In this case, the easiest thing to do is multiply the top equation by 20:

$$(20)(qx+sy=3)$$
$$10x+2y=60$$

$$20qx+20sy=60$$
$$10x+2y=60$$

See, now that both equations, which we know are the same line, are both equal to "60," then the values of the x-coefficients must be the same between the two equations, and likewise for the y-coefficients.

In other words, $20q=10$ and $20s=2$. We can solve for the values of q and s:

$$\frac{20q}{20}=\frac{10}{20}$$
$$q=.5$$

$$\frac{20s}{20}=\frac{2}{20}$$
$$s=.1$$

And now finish by calculating $\frac{q}{s}$:

$$\frac{q}{s}=\frac{.5}{.1}$$
$$=5$$

Note, you could also solve this problem by setting both original equations into $y=mx+b$ form and going from there - a method we've used many times in the past. However, the method I've described above will be somewhat faster, since it was convenient to multiply the top equation by 20 as we did in the first step.

The very *most* important thing to understand is that these two equations are the *same line* and must therefore be identical equations - no matter how you do it.

If you want to be even faster, you can get the final answer by just dividing "10" by "2", which both come from the coefficients of x and y in the bottom equation. However, only very confident math students will be able to draw this conclusion. On the SAT, it's always wise to balance quick insights with careful work to confirm your answer.

43. **B.** This is a Word Problem that also makes use of **Percents**. Here's how I deal with it.

We know the relationship of Jeff's bowl price and Yanik's bowl price is that Yanik's bowl cost $3 less. Therefore:

$$y = j - 3$$

And, the total cost of their meal before tip would be the price of their two bowls added together:

$$y + j = \text{Total before tip}$$

We're supposed to answer in terms of j, so make a convenient substitution of $y = j - 3$:

$$(j-3) + j = \text{Total before tip}$$
$$2j - 3 = \text{Total before tip}$$

Now we need to apply the tip. An 18% tip can be shown by using a multiplier of "1.18", as we learned in the lesson on **Percents**.

Furthermore, it's much smarter to apply the tip to the total meal *before* splitting the cost, instead of splitting the cost and *then* calculating tip:

$$1.18(2j-3) = \text{Total after tip}$$
$$1.18(2j-3) = \text{Total after tip}$$
$$2.36j - 3.54 = \text{Total after tip}$$

Now take the total *after* tip and divide it by 2 to split the meal between the two friends:

$$\frac{2.36j - 3.54}{2} = \text{Cost per friend}$$
$$1.18j - 1.77 = \text{Cost per friend}$$

If the total cost is $1.18j - 1.77$ per person, then the correct answer is Choice B.

44. **A.** Here's another System of Equations question with **Exponents**. First, let's simplify the equation $\frac{n^{x^3}}{n^{y^3}} = n^{15}$,

because we know that exponents raised to other exponents will multiply exponents:

$$\frac{n^{x^3}}{n^{y^3}} = n^{15}$$
$$\frac{n^{3x}}{n^{3y}} = n^{15}$$

Now remember that dividing numbers with the same base but different exponents will *subtract* the bottom exponent from the top exponent:

$$\frac{n^{3x}}{n^{3y}} = n^{15}$$
$$n^{3x-3y} = n^{15}$$

And since the bases n are the same, their exponents must also be equal:

$$n^{3x-3y} = n^{15}$$
$$3x - 3y = 15$$

Remember that the question asks for the value of $x - y$. We can actually just divide our current equation by 3 to finish the question:

$$3x - 3y = 15$$
$$\frac{3x-3y}{3} = \frac{15}{3}$$
$$x - y = 5$$

And there's our final answer: $x - y = 5$, or Choice A.

Notice that it's possible to solve this question without ever using the other given equation $x + y = 7$ - as long as we've mastered our rules of **Exponents** and keep a careful eye out for Algebra shortcuts.

45. **D.** OK, the first step in this question is to FOIL out the left side of the equation (as we learned in **Basic Algebra 2**). As a general rule on SAT Math questions, if you *can* FOIL, you usually should:

$$(ax-3)(bx+6) = 14x^2 + cx - 18$$
$$abx^2 + 6ax - 3bx - 18 = 14x^2 + cx - 18$$

We can add 18 to both sides to simplify things a bit:

$$abx^2 + 6ax - 3bx - 18 = 14x^2 + cx - 18$$
$$+18 \qquad\qquad +18$$
$$abx^2 + 6ax - 3bx = 14x^2 + cx$$

Now, compare the coefficients of the x^2 terms on the left and right side. Since the two sides are balanced, we know that abx^2 must be the same as $14x^2$:

$$abx^2 = 14x^2$$
$$\frac{abx^2}{x^2} = \frac{14x^2}{x^2}$$
$$ab = 14$$

OK, so now we know that $ab = 14$. Put this together with the $a + b = 9$ given by the question and you have a neat little system:

$$a + b = 9$$
$$ab = 14$$

This System can be solved with the "ISS Method," or you can just use your powers of observation to notice that the values "7" and "2" would successfully *multiply* to 14 and *add* to 9. Either way, you'll end up in the same place.

Only two (big) problems: we don't know *which* variable is 2 and which is 7? Have we found that $a = 7$ and $b = 2$, or that $a = 2$ and $b = 7$? We don't know; there's no way to tell.

Furthermore, the question asked for the two possible values of c, not of a and b (I bet a lot of you were already reaching for Choice B!)

So, our work isn't done yet. Here's what we'll do. We're going to find the value of c by plugging our newly-found values of a and b back into the original equation. *BUT*, we're also going to do this *twice* - once in case $a = 7$ and $b = 2$, and a second time in case $a = 2$ and $b = 7$.

Let's consider $a = 7$ and $b = 2$ first. Remember, we're plugging back in and solving for c:

$$(ax-3)(bx+6) = 14x^2 + cx - 18$$
$$(7x-3)(2x+6) = 14x^2 + cx - 18$$

Now FOIL the left side:

$$(7x-3)(2x+6) = 14x^2 + cx - 18$$
$$14x^2 + 42x - 6x - 18 = 14x^2 + cx - 18$$

And combine like terms:

$$14x^2 + 42x - 6x - 18 = 14x^2 + cx - 18$$
$$14x^2 + 36x - 18 = 14x^2 + cx - 18$$

By comparing the two sides of the equation, you can see that the coefficients of x must be the same on both sides. In other words, $36 = c$. It seems like Choice D must be the answer.

Just to be sure, let's finish our plan of swapping the possible values of a and b. We'll repeat the last few steps, this time with $a = 2$ and $b = 7$:

$$(ax-3)(bx+6) = 14x^2 + cx - 18$$
$$(2x-3)(7x+6) = 14x^2 + cx - 18$$

Now FOIL again:

$$(2x-3)(7x+6) = 14x^2 + cx - 18$$
$$14x^2 + 12x - 21x - 18 = 14x^2 + cx - 18$$

And combine like terms:

$$14x^2 + 12x - 21x - 18 = 14x^2 + cx - 18$$
$$14x^2 - 9x - 18 = 14x^2 + cx - 18$$

This time, comparing the two sides of the equation, you can see that the coefficients of x have changed, but must still be the same on both sides. In other words, $-9 = c$, and we've confirmed that Choice D must be the answer.

46. **C.** I've seen a lot of students have difficulty with similar questions on the actual SAT. Here's the key. Let's fill in the table with variables, but let's *not* use a separate variable for each blank space (that would result in *four* unknown variables).

Instead, think about this. Let's use x for Solid Red marbles. Then, there are "three times as many red swirled marbles," or $3x$.

Moving to Blue Marbles, we could use y for Blue Swirled marbles. Then, there are "five times as many blue solid marbles," or $5y$.

We could fill in the table as:

	Pattern	
Color	Solid	Swirled
Red	x	$3x$
Blue	$5y$	y
Total	26	22

So, we've managed to fill in the entire table with only *two* variables. Now let's add up each of the columns to create our System of Equations:

$$x + 5y = 26$$
$$3x + y = 22$$

Great, we're making good progress. Now use the "ISS Method" to solve. It's easiest to Isolate y in the bottom equation:

$$\begin{aligned} 3x + y &= 22 \\ -3x \quad & \quad -3x \\ y &= 22 - 3x \end{aligned}$$

Now we can Substitute into our other equation:

$$\begin{aligned} x + 5y &= 26 \\ x + 5(22 - 3x) &= 26 \end{aligned}$$

Now distribute, combine like terms, and solve for x:

$$\begin{aligned} x + 5(22 - 3x) &= 26 \\ x + 110 - 15x &= 26 \\ -14x + 110 &= 26 \\ -110 \quad &-110 \\ -14x &= -84 \\ \frac{-14x}{-14} &= \frac{-84}{-14} \\ x &= 6 \end{aligned}$$

OK, so $x = 6$. We can update our chart to reflect this:

	Pattern	
Color	Solid	Swirled
Red	6	18
Blue	$5y$	y
Total	26	22

And, from the "Swirled" column, now it's not hard to figure out that y must equal 4:

$$\begin{aligned} 18 + y &= 22 \\ -18 \quad & \quad -18 \\ y &= 4 \end{aligned}$$

And if $y = 4$, then $5y = 20$. We can update our chart again:

	Pattern	
Color	Solid	Swirled
Red	6	18
Blue	20	4
Total	26	22

Notice how everything adds up correctly.

Finally, to answer the question, we're asked for the probability that a randomly-selected swirled marble (22 total possible marbles) is blue (4 possible marbles). Set up a "Desired over Total" fraction, as we learned in the lesson on **Probabilities**.

$$\frac{\text{blue swirled}}{\text{all swirled}} = \frac{4}{22} = .181818...$$

And now we have our final answer, .181 or Choice C.

47. **A.** This is a tough Systems of Equations problem because of its complexity. We have *four* unknown variables: a, b, x, and y. The question provides us two algebraic equations, and one more that we can set up from the words:

$$a = b + \frac{1}{2}$$

Finally, the (correct) answer choice will provide us a fourth and final equation.

Let's just start making moves where we can, and have some faith that things will work out. The first thing we could do is substitute $a = b + \frac{1}{2}$ into the top equation for a:

$$4x - a = 8x - 5$$

$$4x - (b + \frac{1}{2}) = 8x - 5$$

Now distribute, clean up and simplify as much as possible:

$$4x - (b + \frac{1}{2}) = 8x - 5$$

$$4x - b - \frac{1}{2} = 8x - 5$$

$$-4x \qquad\qquad -4x$$

$$-b - \frac{1}{2} = 4x - 5$$

$$+\frac{1}{2} \qquad\quad +\frac{1}{2}$$

$$-b = 4x - 4\frac{1}{2}$$

$$(-1) - b = 4x - 4\frac{1}{2}(-1)$$

$$b = 4\frac{1}{2} - 4x$$

OK, now I have an Isolated version of b. I can Substitute this in for b in the second Algebraic equation from the problem:

$$4y - b = 8y - 5$$

$$4y - (4\frac{1}{2} - 4x) = 8y - 5$$

Now distribute, combine like terms, and simplify:

$$4y - (4\frac{1}{2} - 4x) = 8y - 5$$

$$4y - 4\frac{1}{2} + 4x = 8y - 5$$

$$-4y \qquad\qquad -4y$$

$$-4\frac{1}{2} + 4x = 4y - 5$$

$$+4\frac{1}{2} \qquad\quad +4\frac{1}{2}$$

$$4x = 4y - \frac{1}{2}$$

Keep in mind that all the answer choices start with "x is…" so I'm choosing to solve my algebra for an Isolated x on the left side. Almost done…

$$4x = 4y - \frac{1}{2}$$

$$\frac{4x}{4} = \frac{4y - \frac{1}{2}}{4}$$

$$x = y - \frac{1}{8}$$

And now I've isolated x in terms of y, just like all the answer choices. My final form matches with Choice A.

If you have the patience and precision to work through a question like this one with accuracy and confidence, you're probably ready for every System of Equations question on the SAT Test!