Basic Algebra 1 & Inequalities Answers

1. D
2. D
3. A
4. C
5. B
6. C
7. A
8. A
9. B
10. B

Basic Algebra 1 & Inequalities Explanations

1. **D.** Let's work through this. It's just the bare basics of Algebra.

$$\begin{aligned} 4x - 7 &= 5 - 2x \\ +7 &\quad +7 \\ 4x &= 12 - 2x \\ +2x &\quad +2x \\ \frac{6x}{6} &= \frac{12}{6} \\ x &= 2 \end{aligned}$$

2. **D**. More Basic Algebra.

$$\begin{aligned} -4n - 18 &= -7n + 9 \\ +18 &\quad +18 \\ -4n &= -7n + 27 \\ +7n &\quad +7n \\ \frac{3n}{3} &= \frac{27}{3} \\ n &= 9 \end{aligned}$$

3. **A**. Be sure to Combine Like Terms at the beginning to clean things up before you get started.

$$\begin{aligned} 4x + 4 + 8x + 8 &= 8 + x + 3x - 12 \\ 12x + 12 &= 4x - 4 \\ +4 &\quad +4 \\ 12x + 16 &= 4x \\ -12x &\quad -12x \\ \frac{16}{-8} &= \frac{-8x}{-8} \\ -2 &= x \end{aligned}$$

4. **C**. Notice the "double distribution" on the right side. We always work parentheses from inside to outside (review the Prelesson on **Order of Operations**).

So, do the distribution for the inner parentheses first, then do the distribution for the outer parentheses.

$$\begin{aligned} 4(t + 7) &= 2(3(2 + t)) \\ 4t + 28 &= 2(6 + 3t) \\ 4t + 28 &= 12 + 6t \\ -4t &\quad -4t \\ 28 &= 12 + 2t \\ -12 &\quad -12 \\ \frac{16}{2} &= \frac{2t}{2} \\ 8 &= t \end{aligned}$$

5. **B**. Here's how to work through the Algebra. Notice that my first step is to multiply both sides by 10 to get rid of the fraction. Be sure to distribute it on the right side:

$$\begin{aligned} \frac{-5x - 5}{10} &= 5 - x \\ (10)\frac{-5x - 5}{10} &= (5 - x)(10) \\ -5x - 5 &= 50 - 10x \\ +10x &\quad +10x \\ 5x - 5 &= 50 \\ +5 &\quad +5 \\ \frac{5x}{5} &= \frac{55}{5} \\ x &= 11 \end{aligned}$$

6. **C.** Here's how to work this question. We begin by Combining Like Terms before we do any new work.

$$\frac{21n+18+24n}{4+n}=3(1+5)$$
$$\frac{45n+18}{4+n}=3(6)$$
$$\frac{45n+18}{4+n}=18$$
$$45n+18=18(4+n)$$
$$45n+18=72+18n$$
$$-18 \quad -18$$
$$45n=54+18n$$
$$-18n \qquad -18n$$
$$27n=54$$
$$\frac{27n}{27}=\frac{54}{27}$$
$$n=2$$
$$3n=6$$

Notice the Switcheroo in the question asking for "$3n$", not "n". The fake answer choice A was a total setup: the first thing you see when you get "$n=2$" is Choice A for "2". Review the Prelesson on **Careless Mistakes** if you fell for this trap!

7. **A.** Here's how to work this inequality:

$$y+7<2y+4$$
$$-y \qquad -y$$
$$7<y+4$$
$$-4 \qquad -4$$
$$3<y$$

Notice that y can be any value greater than 3, which leaves us with Choice A as the only option.

If you get Choice D, it's possible you solved with a different order of solution steps. In that case, you may have forgotten to flip the direction of the inequality if you multiplied or divided both sides by a negative number.

8. **A.** Here's how to work through this inequality.

$$7-x<\frac{-4x-2}{2}$$
$$2(7-x)<-4x-2$$
$$14-2x<-4x-2$$
$$+2 \qquad\qquad +2$$
$$16-2x<-4x$$
$$+2x \quad +2x$$
$$\frac{16}{-2}<\frac{-2x}{-2}$$
$$-8>x$$

Notice the inequality flips direction in the last step because we divide both sides by a negative number. Because x must be *less* than -8, only Choice A works.

9. **B**. Here's something new: inequalities on both sides of our equation. The good news is everything works exactly the same as before, except we have three "sides" to balance instead of two. It's easy, though. Follow my work:.

$$1-x<5+3x<17-x$$
$$+x \qquad +x \qquad +x$$
$$1<5+4x<17$$
$$-5 \; -5 \qquad -5$$
$$\frac{-4}{4}<\frac{4x}{4}<\frac{12}{4}$$
$$-1<x<3$$

This resulting inequality "sandwich" tells us that x can be any number more than -1 and less than 3.

10. **B**. This is another "inequality sandwich" question, and we deal with it the same way as Question 9. Focus on Combining Like Terms and "capturing" the n in the middle of the inequality. Here's one way to solve it:

$$2(6n-4)\le 4(4+2n)\le 6(2+2n)$$
$$12n-8\le 16+8n\le 12+12n$$
$$-12n \qquad -12n \qquad -12n$$
$$-8\le 16-4n\le 12$$
$$-16 \; -16 \qquad -16$$
$$\frac{-24}{-4}\le\frac{-4n}{-4}\le\frac{-4}{-4}$$
$$6\ge n\ge 1$$

Be careful to catch both inequality signs flipping direction at the end, when you divide all three "sides" by -4.

Advanced Algebra 1 Answers

1. C
2. B
3. B
4. D
5. A
6. C
7. A
8. B
9. C
10. D

Advanced Algebra 1 Explanations

1. **C.** Start this question with cross-multiplying. Then distribute and combine like terms.

$$\frac{2.5x+3.5}{4}=\frac{1.25-2.25x}{2}$$
$$2(2.5x+3.5)=4(1.25-2.25x)$$
$$5x+7=5+9x$$
$$-5x \qquad -5x$$
$$7=5+4x$$
$$-2-2$$
$$\frac{2}{4}=\frac{4x}{4}$$
$$\frac{1}{2}=x$$

And of course, $\frac{1}{2}=.5$.

2. **B.** Barely any work to do here - just divide both sides by 5!

$$\frac{10a-25b}{5}>\frac{25}{5}$$
$$2a-5b>5$$

3. **B.** This has all the signs of a "Square Roots & False Solutions" question. So, just test the answer choices:

$x=-1$ is a successful answer choice:

$$\sqrt{3(-1)+4}=-(-1)$$
$$\sqrt{-3+4}=1$$
$$\sqrt{1}=1$$
$$1=1$$

$x=4$ is NOT a successful answer choice:

$$\sqrt{3(4)+4}=-(4)$$
$$\sqrt{12+4}=-4$$
$$\sqrt{16}=-4$$
$$4\neq-4$$

4. **D.** The first thing to do is plug in the given starting values of $y=4$ and $x=2$, then solve for a, like this:

$$y=ax$$
$$4=2a$$
$$\frac{4}{2}=\frac{2a}{2}$$
$$2=a$$

The next thing to do is set up the equation again, this time knowing that the constant a is always 2.

$$y=ax$$
$$y=2x$$

Then plug in the new value, $x=4$:

$$y=2x$$
$$y=2(4)$$
$$y=8$$

Finally, avoid falling for a Switcheroo. They're asking for the value of $y+2$, NOT the value of y. Since $y=8$ the value of $y+2=10$.

5. **A.** Try making up your own values. The question says "which of the following *must* be true," so any values should work, as long as they fit the descriptions of positive and negative numbers in the question.

Let's use a positive value for t, like 6, and a negative value for z, like -2.

$$\frac{t-z}{z}=q$$
$$\frac{6-(-2)}{-2}=q$$
$$\frac{6+2}{-2}=q$$
$$\frac{8}{-2}=q$$
$$-4=q$$

Notice that the value of q is a negative, so we can safely assume that $q<0$. Don't believe me? Try using any positive value for t and any negative value for z. You'll always get a negative value for q.

6. **C.** This has all the signs of a "Square Roots & False Solutions" question. Simply test the answer choices, just like we did in Question 3.

First test $x = -5$:

$$\sqrt{10-3(-5)} = -5$$
$$\sqrt{10+15} = -5$$
$$\sqrt{25} = -5$$
$$5 \neq -5$$

Note that "5" does NOT equal "-5", so this is a false solution.

Then test $x = 2$:

$$\sqrt{10-3(2)} = 2$$
$$\sqrt{10-6} = 2$$
$$\sqrt{4} = 2$$
$$2 = 2$$

This creates a true equality, so $x = 2$ is the only valid solution for this equation.

7. **A.** This is a "Word Problem with Rearrangement" question, just like we explored earlier in the chapter. In these cases, remember that understanding the word problem itself is not necessary to finish the problem.

All of the answer choices show us that our only job is to isolate b on the left side of the equation. That's easier than it looks - follow my steps, starting with the original equation:

$$m = \frac{h(h^3+l)}{b\sqrt{2h+2l}}$$
$$(b)m = \frac{h(h^3+l)}{b\sqrt{2h+2l}}(b)$$
$$bm = \frac{h(h^3+l)}{\sqrt{2h+2l}}$$
$$(\frac{1}{m})bm = \frac{h(h^3+l)}{\sqrt{2h+2l}}(\frac{1}{m})$$
$$b = \frac{h(h^3+l)}{m\sqrt{2h+2l}}$$

8. **B.** This question tests our Cross-Multiplying skills, and also requires us to find the combined variable fraction (or "ratio") of $\frac{y}{x}$, instead of just finding the value of x or y by themselves.

Here are the steps:

$$\frac{y-2}{2} = \frac{x-8}{8}$$
$$8(y-2) = 2(x-8)$$
$$8y-16 = 2x-16$$
$$+16 \qquad +16$$
$$8y = 2x$$
$$\frac{8y}{x} = \frac{2x}{x}$$
$$8(\frac{y}{x}) = 2$$
$$(\frac{1}{8})8(\frac{y}{x}) = 2(\frac{1}{8})$$
$$\frac{y}{x} = \frac{2}{8}$$
$$\frac{y}{x} = \frac{1}{4}$$

Don't forget to add "+1" at the end since the question asks for $\frac{y}{x}+1$, to avoid the sneaky Switcheroo mistake they're trying to cause.

$$\frac{y}{x}+1$$
$$=\frac{1}{4}+1$$
$$=1.25$$

9. **C.** This is another "Word Problem with Rearrangement" question, which means the answer choices reveal what constant or variable to isolate, then we ignore the rest of the question since it's only meant to distract us.

In this case, the answer choices show us that we're meant to isolate x. Let's do it! Starting with the original equation, focus on getting x by itself:

$$v = \frac{\sqrt{g-n^2}+17n}{3x-12n}$$

$$(3x-12n)v = \frac{\sqrt{g-n^2}+17n}{3x-12n}(3x-12n)$$

$$3xv-12nv = \sqrt{g-n^2}+17n$$

$$+12nv \qquad\qquad +12nv$$

$$3xv = \sqrt{g-n^2}+17n+12nv$$

$$\frac{3xv}{3v} = \frac{\sqrt{g-n^2}+17n+12nv}{3v}$$

$$x = \frac{\sqrt{g-n^2}+17n+12nv}{3v}$$

In addition to matching the correct answer to your final equation, I also recommend eliminating some of the answer choices that don't work. For example, several of the answer choices have negative signs in the wrong places This reduces how much mental processing you need to do and reduces careless errors.

10. **D.** Here's one last "Word Problem with Rearrangement" question. We've seen a lot of these by now. This one looks particularly ugly, so keep your eyes on the prize: we're supposed to isolate D on one side.

Also, view this ugly fraction as a "big fraction" with a "smaller fraction" on the bottom of it. It may help to review the Prelesson on **Fractions**.

Notice in the last step of my solution, we flip both the left and right sides upside down at the same time.

Here are the moves, starting with the original equation.

$$v = \frac{D\sqrt{pn^2+100i}}{i(\frac{1-\sqrt{i-n^3}}{p-20i})}$$

$$(\frac{1}{D})v = \frac{D\sqrt{pn^2+100i}}{i(\frac{1-\sqrt{i-n^3}}{p-20i})}(\frac{1}{D})$$

$$\frac{v}{D} = \frac{\sqrt{pn^2+100i}}{i(\frac{1-\sqrt{i-n^3}}{p-20i})}$$

$$(\frac{1}{v})\frac{v}{D} = \frac{\sqrt{pn^2+100i}}{i(\frac{1-\sqrt{i-n^3}}{p-20i})}(\frac{1}{v})$$

$$\frac{1}{D} = \frac{\sqrt{pn^2+100i}}{vi(\frac{1-\sqrt{i-n^3}}{p-20i})}$$

$$D = \frac{vi(\frac{1-\sqrt{i-n^3}}{p-20i})}{\sqrt{pn^2+100i}}$$

Absolute Value Answers

1. B
2. D
3. 5
4. C
5. D
6. 2
7. C
8. B
9. A
10. 25

Absolute Value Explanations

1. **B.** This problem gives us a Basic Algebra problem combined with Absolute Value. Remember to set up both the "positive side" and "negative side" solutions, as we learned in this lesson. That means considering the possibility that the equation should equal 10 *or* -10:

$$\begin{aligned} x-6&=10 \\ +6\ \ &\ \ +6 \\ x&=16 \end{aligned} \qquad\qquad \begin{aligned} x-6&=-10 \\ +6\ \ &\ \ +6 \\ x&=-4 \end{aligned}$$

There are two solutions for *x*, but the question asked us for a value of *x* that is less than zero. Our only option is $x=-4$.

2. **B.** Another Basic Algebra problem combined with Absolute Value. As we learned in the lesson on **Basic Algebra 1**, it's best to Combine Like Terms before doing any more complicated steps. We'll do that first:

$$\begin{aligned} 3+|n|&=12 \\ -3\qquad &\ \ -3 \\ |n|&=9 \end{aligned}$$

Now that the basic equation is cleaned up, consider both the "positive side" and "negative side" solutions. There's not much to do, though. If $|n|=9$ then $n=9$ or $n=-9$.

3. **5.** More Basic Algebra combined with Absolute Values. Remember to set up and solve for *both* the "positive side" and "negative side" solutions (both $+14$ and -14 on the right side).

Also, watch your negative signs. Just because $-4-2t$ is inside Absolute Value bars does *not* mean that all those negative signs and subtraction magically disappear. Absolute Value bars act like parentheses - whatever takes place within them still needs to happen first, regardless of whether the terms are positive or negative.

$$\begin{aligned} -4-2t&=14 \\ +4\qquad &\ \ +4 \\ \frac{-2t}{-2}&=\frac{18}{-2} \\ t&=-9 \end{aligned} \qquad\qquad \begin{aligned} -4-2t&=-14 \\ +4\qquad &\ \ +4 \\ \frac{-2t}{-2}&=\frac{-10}{-2} \\ t&=5 \end{aligned}$$

This Free Response question asks us for a value of t that is greater than zero. The only possibility is the value $t=5$.

4. **C.** More **Basic Algebra 1** combined with Absolute Values. The first thing to do is clean it up: the absolute value of "-9" is always 9, so we can simplify the right side of the equation as $|-2x+3|=9$.

Then solve for both the "positive side" and the "negative side" of the equation, just as we've done in the previous three problems.

$$\begin{aligned} -2x+3&=9 \\ -3\ &\ -3 \\ \frac{-2x}{-2}&=\frac{6}{-2} \\ x&=-3 \end{aligned} \qquad\qquad \begin{aligned} -2x+3&=-9 \\ -3\ &\ -3 \\ \frac{-2x}{-2}&=\frac{-12}{-2} \\ x&=6 \end{aligned}$$

Our solution set is both values: $x=\{-3,6\}$

5. **D.** You guessed it, more **Basic Algebra 1** with Absolute Value. Start off by setting up the "positive side" and "negative side" of the equation.

$$x - 4.5 = 3.5 \qquad\qquad x - 4.5 = -3.5$$
$$+4.5 \ +4.5 \qquad\qquad +4.5 \quad +4.5$$
$$x = 8 \qquad\qquad x = 1$$

Remember that the question asks $a + b$, where a and b are the solutions of the equation. Finish the question with "$8+1$", giving a final value of 9.

6. **2.** More **Basic Algebra 1** with Absolute Value. First let's run through the essentials: set up both the "positive side" and "negative side" of the equation:

$$4x + 12 = 4 \qquad\qquad 4x + 12 = -4$$
$$-12 \ -12 \qquad\qquad -12 \quad -12$$
$$\frac{4x}{4} = \frac{-8}{4} \qquad\qquad \frac{4x}{4} = \frac{-16}{4}$$
$$x = -2 \qquad\qquad x = -4$$

But, we're not done yet. The question asked for the value of $|n - t|$, where n and t are solutions to the equation (we know the solutions are "-2" and "-4"). Finish it off:

$$|-2 - (-4)|$$
$$|-2 + 4|$$
$$|2| = 2$$

Notice that it doesn't matter which solution is which. Whether you assign -2 or -4 to n or t, you'll get the same final result. Don't believe me? Try both ways yourself!

7. **C.** This is obviously an Absolute Value graph because of it's V-shape (looks like "a laser reflecting off a mirror"). Therefore we can eliminate Choices A and D, neither of which involve an Absolute Value.

But, we're left with Choices B and C. How to tell them apart? Well, Choice B has a "-4" *outside* the Absolute Value bars, meaning that the graph of that equation should still dip below $y = 0$. However, our graph never passes below the *x*-axis.

That leaves us only with the correct answer, Choice C, which has the entire equation contained inside Absolute Value bars - thereby preventing the graph from ever dropping into negative *y*-value territory.

8. **B.** This is an interesting question! We're asked to consider a number line. I suggest your first move is to sketch one. Just make a medium-length straight horizontal line with arrows at both ends, then mark a point near the center with the coordinate "$x = -2$". Here's an example (forgive my lack of artistry):

<---------- (-2) ------------->

OK, now count 7 units up from -2 (otherwise known as $-2+7$). This gives you a point at $x = 5$. Then count 7 units *down* from -2 (or just calculate $-2-7$). This gives you another point at $x = -9$. Here's our number line with the new values filled in:

<- (-9) -------- (-2) -------- (5) ---->

The easiest option at this point is just to test all four multiple-choice answers. You should be able to plug in *either* "-9" or "5" and get a true and balanced equation both times. Only Choice B will succeed for both values.

Alternately - if you're feeling more confident - you can work this a quicker way. The concept of "distance" is also a concept of "difference," and "difference" is just another word for "subtraction." Take any two coordinate points, subtract one from the other, and take the Absolute Value of the result - you'll have the distance between the two points.

We know the point "-2" is our first coordinate, and we can use the variable x to represent our other unknown coordinates.

Follow the instructions above and you can set up this equation: $|x-(-2)| = 7$. In other words, the distance of point x from -2 is 7 units. This equation can be simplified to the correct answer: $|x+2| = 7$.

9. **A.** As with the other Graphing questions in this lesson, you first should recognize the characteristic shape of an Absolute Value graph. However, something's different about this one: it's facing upside-down!

What's the reason for this? Well, this is what happens when you apply a negative sign to the *outside* of the Absolute Value bars that contain the x variable. It inverts ("turns upside-down" the shape of the graph).

Think of it like this: the Absolute Value "forces" the value of x to come out as a positive, but then the negative sign on the outside "forces" those same positive values to become negative. That negative sign on the outside of the Absolute Value bars is how you end up with an Absolute-Value-shaped graph that heads *downwards* instead of upwards. (This also relates to upcoming lessons on **Linear Equations**).

None of other choices (B, C, and D) have a negative sign on the outside of the Absolute Value, so their "V-shapes" would all face upwards, and cannot be the right answers.

10. **25.** Another Absolute Value Number Line question. Call me crazy, but I really enjoy these problems.

Best thing to do is probably to sketch out a number line again, as in our solution to Question 8, for the helpful visual cues.

Then you'll need to both add 12.5 and subtract -12.5 from the starting coordinate of -6. That gives $-6+12.5 = 6.5$ and $-6-12.5 = -18.5$. Now we know our values of s and t: 6.5 and 18.5.

Finish the question by completing the final expression $|s-t|$:

$$
\begin{aligned}
&|6.5-(-18.5)| \\
&|6.5+18.5| \\
&=|25| \\
&=25
\end{aligned}
$$

Again, it doesn't matter which value is assigned to which value - you'll get the same answer either way. As always, if you don't believe me, just try it for yourself (it's a good learning exercise!)

Algebra 1 Word Problems Answers

1. D
2. C
3. B
4. 20
5. 15
6. B
7. D
8. C
9. A
10. B
11. A
12. B
13. 6
14. D
15. A
16. D
17. C
18. D
19. B
20. 1.5

Algebra 1 Word Problems Explanations

1. **D.** This is just a basic "Translating Words into Algebra" question. Set it up and solve as follows:

$$4q - 15 = 26 + 19$$
$$4q - 15 = 45$$
$$+15 \quad +15$$
$$\frac{4q}{4} = \frac{60}{4}$$
$$q = 15$$

Now we know $q = 15$, but don't forget to finish the question, which asked for the value of $3q$.

$$3(15) = 45$$

2. **C.** Another "Translating Words into Algebra" question like the previous. Set it up and solve as follows:

$$21 + 7x = 68 - 12$$
$$21 + 7x = 56$$
$$-21 \quad\quad -21$$
$$\frac{7x}{7} = \frac{35}{7}$$
$$x = 5$$

But we're not done yet. Although $x = 5$, we've also been asked "how many times does $4x$ divide into 60?"

Since $4x = 4(5) = 20$, we can ask "how many times does 20 divide into 60?" The answer is $\frac{60}{20} = 3$.

3. **B.** It's a pretty simple word problem - don't overthink it. Each pint will cost \$3, and there will be r pints. It costs three more dollars for each additional pint. That means our total price equation will just be \$3 per r, or $3r$.

4. **20.** This word problem is about using the provided equation and plugging the correct numbers into the proper variables, then solving. The cost of manufacturing, T, is 2000. The value of c is given as 10 speaker cabinets.

Here's our setup and solution:

$$T = 100c + 50s$$
$$2000 = 100(10) + 50s$$

Now solve for s:

$$2000 = 100(10) + 50s$$
$$2000 = 1000 + 50s$$
$$-1000 \quad -1000$$
$$\frac{1000}{50} = \frac{50s}{50}$$
$$20 = s$$

So the value of s, the number of speaker cones, is 20.

5. **15.** The first thing we need to do with this Word Problem is turn it into an Algebra equation. Let's make up our own variable T to represent the total cost of membership and use x to represent the number of months we sign up for. We'll have to pay our \$50 sign-up fee and m dollars per month. Here's what that equation looks like:

$$T = 50 + mx$$

Now let's plug in the values we are given in the word problem: \$230 is our total T value, and 12 is our x value. We're solving for m.

$$230 = 50 + 12m$$
$$-50 \quad -50$$
$$180 = 12m$$
$$\frac{180}{12} = \frac{12m}{12}$$
$$15 = m$$

6. **B.** This Word Problem is about distance. As I mentioned in the lesson, distance is "the absolute value of subtraction." Another way of saying this is "distance is the difference between two points, and it is always a positive value."

We could use Absolute Value to solve this problem in one giant setup, but I think it's overkill. Instead, just go a step at a time and write down each stage of his journey. From home at Mile Marker 6 to Mile Marker 5 is $|6-5|=$ *1 mile*. Then from Mile Marker 5 to Mile Marker 9 is $|5-9|=$ *4 miles*. From Mile Marker 9 to Mile Marker 3 is $|9-3|=$ *6 miles*. From Mile Marker 3 to home at Mile Marker 6 is $|3-6|=$ *3 miles*.

Total up all the miles he's walked and you'll have $1+4+6+3=14$ miles.

7. **D.** Use the word problem to create a setup. I'm going to use decimals instead of fractions to make my life easier - it's just a personal preference. The decimal value of $4\frac{2}{5}$ inches is 4.4 inches. I've also chosen a new variable, A, to represent the height of a single individual ant.

$$40A = 4.4$$
$$\frac{40A}{40} = \frac{4.4}{40}$$
$$A = .11$$

Now I know an individual ant has a height of .11 inches, and I can set up another equation for the 10-inch column. This time I'll use the variable x to represent the number of ants:

$$10 = .11x$$
$$\frac{10}{.11} = \frac{.11x}{.11}$$
$$90.9... = x$$

Although I get an ugly repeating decimal for the value of x, I've truncated at "90.9" because we're asked to round to the "closest number," which will be 91 ants.

8. **C.** This question asks us to interpret the real-world meaning of the terms in an Algebra equation.

Here's how I see it. On the right side of the equation we have two terms being totaled (added) to each other. It's reasonable to assume that one of those terms is Christian's walking time and the other term is his jogging time. In that case, "300" would be the *total* of all Christian's time spent jogging and walking.

The only other issue is the units, because the word problem uses "minutes" for r and "hours" for w. But notice that w is being multiplied by 60, which is exactly how we would convert w hours into an equivalent time in minutes. Therefore, it's reasonable to assume that "300" is the *total* number of *minutes* spent jogging and walking each week.

9. **A.** This Word Problem provides us an Algebra equation to use. It can be a bit confusing to find the useful data in all those words, but notice the "10 liters of the 25% by volume solution" near the end. 10 is a value we can plug in for the variable y (be sure to plug it in for y on *both* the left and the right side of the equation).

$$.15x + .25y = .2(x+y)$$
$$.15x + .25(10) = .2(x+10)$$

That creates a complete Algebra setup that we can solve, as follows:

$$.15x + .25(10) = .2(x+10)$$
$$.15x + 2.5 = .2x + 2$$
$$-2 \qquad\qquad -2$$
$$.15x + .5 = .2x$$
$$-.15x \qquad -.15x$$
$$\frac{.5}{.05} = \frac{.05x}{.05}$$
$$10 = x$$

The value of x, the liters of 15% by volume solution, is 10.

10. **B.** This Word Problem gives us the opportunity to set up an Algebra inequality for Ian's purchase. I'll make up the variable g to represent the number of video games, and c to represent the number of controllers:

$$225 > 50g + 30c$$

Notice my use of an inequality to show that Ian's total payment must be *less than* his maximum budget of $225.

Now plug in the given value of "3 video games" for g and solve the inequality:

$$\begin{aligned} 225 &> 50g + 30c \\ 225 &> 50(3) + 30c \\ 225 &> 150 + 30c \\ -150 & \quad -150 \\ 75 &> 30c \end{aligned}$$

Notice that I've stopped right before I finish. Why? Because I'm not allowed to use my calculator, I don't want to actually divide 75 by 30. However, we can easily calculate that 2 controllers would cost $60 and remain within budget, but 3 controllers would cost $90 and go over-budget. Therefore, Ian cannot buy more than 2 controllers with his current budget.

11. **A.** This word problem provides us with an equation - we simply have to plug in the given values and solve. The value of r is 110 race car tires, as given in the question. Plug in and solve for t:

$$\begin{aligned} 2520 &= 8r + 14t \\ 2520 &= 8(110) + 14t \\ 2520 &= 880 + 14t \\ -880 & \quad -880 \\ \frac{1640}{14} &= \frac{14t}{14} \\ 117.14... &= t \end{aligned}$$

Note that we get an ugly decimal value for t. That's OK - the question asked for the *maximum* number of tractor tires t. Of course, there's no such thing as ".14 of a tire", so we must round down to 117.

12. **B.** This Word Problem requires that we translate the words into an Algebra setup with an inequality. We want to find the length of time it will take for the factory to break even on their new $20,000 investment. Therefore, we can start with the following:

$$20000 < \text{amount saved}$$

This inequality says we're looking for the moment when the amount saved is *greater* than the $20,000 investment.

Now we just need to set up the right side of the equation. The factory produces c canisters per month and each canister produced will save $1.50. We'll also need to multiply by the number of months of production. Let's fill this information in:

$$20000 < \$1.5c(\text{months of production})$$

We're told to use the variable y for months of production, so we can now finish our setup, and we're done!

$$20,000 < 1.5cy$$

13. **6.** This question isn't so bad. We translate the words into an equation. I'll make up some common-sense variables: m will be "mass," v will be "volume," and d will be "density."

Here's what I come up with from turning the word problem into an equation:

$$d = \frac{m}{v}$$

Now I just plug in the given values from the word problem and solve:

$$\begin{aligned} 9 &= \frac{54}{v} \\ (v)9 &= \frac{54}{v}(v) \\ 9v &= 54 \\ \frac{9v}{9} &= \frac{54}{9} \\ v &= 6 \end{aligned}$$

14. **D.** This Word Problem asks us to set up an inequality representing the timeframe for completing a project.

A key concept is staying "within 5 hours of the actual time." Another way of saying this is: "the *difference* between the *estimated* time and the *actua*l project time must be less than 5 hours." Remember that "difference" means subtraction, so $x-h$ could represent the difference between actual time and estimated time.

But, there's an important point to make: the question didn't state whether we needed to be *over* or *under* the estimate. We need to account for both possibilities. That means either "-5" or "$+5$" can be the difference between our two times. This is why the correct answer has an inequality on both sides of the $x-h$, which covers both "5 hours over estimate" and "5 hours under estimate."

Note that we never need to use the $h<30$ information for anything, which is unusual, but sometimes happens.

Also understand that either $x-h$ or $h-x$ will work. Both will calculate the difference (subtraction) between the estimated time and the actual completion time. Either way, the difference must be between -5 and 5 hours. This could also be set up with an **Absolute Value**.

15. **A.** This question seems so easy, but I've noticed a lot of my students have a surprising amount of difficulty with it.

One way to deal with it is to simply Make Up Numbers and plug them in. For example, what if we use just one pinch of cheese - in other words, if $g=1$? Plugging it into the equation would produce $s=4$, or four servings of omelets. In other words, 1 pinch of cheese will make 4 servings.

Now increase g by one, to $g=2$, and plug it in. The result is $s=5$. Notice that by adding 1 pinch of cheese, we got one additional serving of omelets. That's one way to see that for each additional serving, you need one more pinch of cheese.

Another way to view this question is as a Linear Equation of the form $y=mx+b$. From that point of view, you can see that the slope of this line is "1", which means the pinches of cheese and servings of omelette will increase together at a constant 1-to-1 relationship.

16. **D.** This first problem in the set of three is the "gimme" problem - the easiest out of all three "Skyscraper" questions. All of the answer choices show us that we should solve for h. Just take the original equation, $4h+p=170$, and do **Basic Algebra 1** to isolate the variable h. Here are the steps:

$$\begin{aligned} 4h+p&=170 \\ -p \quad & \quad -p \\ 4h&=170-p \\ \frac{4h}{4}&=\frac{170-p}{4} \\ h&=\frac{170-p}{4} \end{aligned}$$

17. **C.** This question is somewhat more difficult. We must create an inequality for all the possible values of h.

First, the easy part: at a minimum, h must be "no less than 10 feet," or $10<h$. Unfortunately, that doesn't eliminate any answer choices. We still need to find the maximum value of h.

Now, the harder part. We need to use the value "80" that was given in the word problem, and plug it in to the correct variable (p for perimeter of the base). Then solve for h. I will reuse our rearranged equation from Question 16 to save time, but we could also use the original equation from the question:

$$\begin{aligned} h&=\frac{170-p}{4} \\ h&=\frac{170-(80)}{4} \\ h&=\frac{90}{4} \\ h&=22.5 \end{aligned}$$

This solution, $h=22.5$,is the value of the floor height when the base perimeter is at its *minimum* value of 80 feet. Remember that "if you want to *maximize* one variable, you must *minimize* all the other variables." And that's what we've just done.

Now we know the floor height cannot exceed a maximum of 22.5 feet without violating the minimum base perimeter requirement.

Our final inequality can be set up as $10<h<22.5$.

18. **D.** This is the hardest question (by far) from the set of three "skyscraper" questions. Where to even begin? Well, of all the values in this word problem, the *only* given values that can plug directly into our equation are the base perimeter requirements: "at least 86 feet but no more than 90 feet."

What I recommend doing is calculating the minimum and maximum floor heights by using *both* values of 86 and 90. Yes, you'll have to work through the equation two times. Note, I'll use the rearranged equation from Question 17 just to save a couple of steps, but you could also use the original equation from the word problem:

$$h_1 = \frac{170-p}{4} \qquad h_2 = \frac{170-p}{4}$$

$$h_1 = \frac{170-(86)}{4} \qquad h_2 = \frac{170-(90)}{4}$$

$$h_1 = \frac{84}{4} \qquad h_2 = \frac{80}{4}$$

$$h_1 = 21 \qquad h_2 = 20$$

Taken together, these two h values tell us the minimum floor height for this building is 20 feet, and the maximum is 21 feet. What we will do next is divide the total height of the skyscraper (given in the question as 780 feet) by *both* possible floor heights. This will tell us the minimum and maximum number of *floors* to build this building.

$$\frac{780}{20} = 39 \qquad \frac{780}{21} = 37.14...$$

Now we know that the skyscraper must have between a minimum of "37.14" floors and a max of 39 floors. The word problem also tells us that the skyscraper must have an odd number of floors. Also, floors must be a whole number (naturally, since you can't have "part of a floor" - you either have one, or you don't). Since 37.14 is a *minimum*, we can't round down to 37. The next option, 38 floors, is not an odd number. Only 39 floors is an odd number of floors that fits within our minimum and maximum number of floors.

Now we can take our 780 foot skyscraper and divide it by the 39 floors to determine how tall each floor must be in feet:

$$\frac{\text{780 feet tall building}}{\text{39 floors}} = \text{20 feet per floor}$$

Almost done. Now plug the "20 feet per floor" back into the original equation for h and solve for p:

$$\begin{aligned} 4h + p &= 170 \\ 4(20) + p &= 170 \\ 80 + p &= 170 \\ -80 \quad & \quad -80 \\ p &= 90 \end{aligned}$$

Voila! We have finally determined that the base perimeter of the skyscraper must be 90 feet, which also fits within the original constraints of the project ("no less than 86 feet but no more than 90 feet).

19. **B.** This question might look like an Algebra setup, but I actually find it easier to solve if I just *think* about it. After giving out 4 pieces of candy to each friend the first time, Aaron has 7 pieces of candy left over. Now he wants to give each friend *one more* piece of candy (going from 4 per friend to 5 per friend). To do this, we know he'll need another 6 candies.

Combine the 7 leftover pieces he already has with the additional 6 pieces he needs, and you'll get 13 pieces. Once Aaron has those 13 pieces of candy, he's able to give everyone exactly one more piece of candy. Therefore, he must have 13 friends at the party - one extra piece of candy per friend.

20. **1.5.** The final question of this section - and it's a doozy. This will get pretty "mathy," so hold on to your hats. First of all, all three celestial bodies have "exactly equal masses". That's good news - it means we can replace m_1, m_2 etc. with just one identical variable, m.

Now let's set up two separate equations - one for the "closer" gravitational force, and one for the "farther" force. Notice my use of "sub-labels" to keep track of my variables:

$$F_{\text{closer}} = G\frac{(m)(m)}{(d_{\text{closer}})^2}$$

$$F_{\text{farther}} = G\frac{(m)(m)}{(d_{\text{farther}})^2}$$

We also know that the number "2.25" must be important. It represents how many times greater F_{closer} is than F_{farther}. The "closer" force is stronger, so this 2.25 multiplier goes on the "farther" side to help it balance the scales. We can write this mathematically:

$$F_{\text{closer}} = 2.25(F_{\text{farther}})$$

Now let's put all these equations together. Plug the first two equations into their respective places in the third equation that we just made:

$$F_{\text{closer}} = 2.25(F_{\text{farther}})$$
$$G\frac{(m)(m)}{(d_{\text{closer}})^2} = 2.25[G\frac{(m)(m)}{(d_{\text{farther}})^2}]$$
$$G\frac{(m)(m)}{(d_{\text{closer}})^2} = 2.25G\frac{(m)(m)}{(d_{\text{farther}})^2}$$

This looks ugly, but a lot of terms can cancel. The G constants can be divided out of both sides. So can the $(m)(m)$ terms on top of both sides. Now we're left with this:

$$(1)\frac{(1)(1)}{(d_{\text{closer}})^2} = 2.25(1)\frac{(1)(1)}{(d_{\text{farther}})^2}$$
$$\frac{1}{(d_{\text{closer}})^2} = 2.25\frac{1}{(d_{\text{farther}})^2}$$
$$\frac{1}{(d_{\text{closer}})^2} = \frac{2.25}{(d_{\text{farther}})^2}$$

From here lets' square root both sides to cancel the exponents,

$$\frac{1}{(d_{\text{closer}})^2} = \frac{2.25}{(d_{\text{farther}})^2}$$
$$\sqrt{\frac{1}{(d_{\text{closer}})^2}} = \sqrt{\frac{2.25}{(d_{\text{farther}})^2}}$$
$$\frac{\sqrt{1}}{\sqrt{(d_{\text{closer}})^2}} = \frac{\sqrt{2.25}}{\sqrt{(d_{\text{farther}})^2}}$$
$$\frac{1}{d_{\text{closer}}} = \frac{1.5}{d_{\text{farther}}}$$

I used my calculator to determine the value of $\sqrt{2.25}$, by the way. It looks much cleaner now, right?

Now cross-multiply, then rearrange into a ratio ("fraction") of the farther distance to the closer distance (which can be expressed as $\frac{d_{\text{farther}}}{d_{\text{closer}}}$):

$$\frac{1}{d_{\text{closer}}} = \frac{1.5}{d_{\text{farther}}}$$
$$1.5(d_{\text{closer}}) = 1(d_{\text{farther}})$$
$$\frac{1.5(d_{\text{closer}})}{(d_{\text{closer}})} = \frac{d_{\text{farther}}}{(d_{\text{closer}})}$$
$$1.5 = \frac{d_{\text{farther}}}{d_{\text{closer}}}$$

We have finally finished, demonstrating that the ratio of the farther distance to the closer distance is 1.5.